University of Southern Queensland
Faculty of Health, Engineering and Sciences

TOPOLOGY OPTIMISATION OF LARGE REINFORCED CONCRETE BOX CULVERTS UNDER SM1600 LOADS

A dissertation submitted by

Mrs. Juliana Sweeney

In fulfilment of the requirements for the degree of

Bachelor of Engineering Hons. (Civil)

November 2014

ABSTRACT

Topology optimisation of large reinforced concrete box culverts under SM1600 loads

by Juliana Sweeney

Supervisor: Dr. Sourish Banerjee
School of Civil Engineering and Surveying

This research project is concerned with finding the optimum three-sided large box culvert through topology optimisation using finite element analysis. The objective function is to minimise the total strain energy while the design constraints include minimising volume as a fraction of the initial volume and geometric restrictions to ensure symmetry and appropriate cover to reinforcement. The optimised culvert must also comply with the latest Australian specifications, must be subjected to standard SM1600 loads for main roads and must be feasible and constructible to be useful and practical to the Australian industry.

LIMITATIONS OF USE

The Council of the University of Southern Queensland, its Faculty of Health, Engineering & Sciences, and the staff of the University of Southern Queensland, do not accept any responsibility for the truth, accuracy or completeness of material contained within or associated with this dissertation.

Persons using all or any part of this material do so at their own risk, and not at the risk of the Council of the University of Southern Queensland, its Faculty of Health, Engineering & Sciences or the staff of the University of Southern Queensland.

This dissertation reports an educational exercise and has no purpose or validity beyond this exercise. The sole purpose of the course pair entitled "Research Project" is to contribute to the overall education within the student's chosen degree program. This document, the associated hardware, software, drawings, and other material set out in the associated appendices should not be used for any other purpose: if they are so used, it is entirely at the risk of the user.

CERTIFICATION OF DISSERTATION

I certify that the ideas, designs and experimental work, results, analyses and conclusions set out in this dissertation are entirely my own effort, except where otherwise indicated and acknowledged.

I further certify that the work is original and has not been previously submitted for assessment in any other course or institution, except where specifically stated.

Juliana Sweeney
0061027945

ACKNOWLEDGMENTS

The author wishes to express sincere appreciation to Dr. Sourish Banerjee for his help and level of commitment to supervising this project.

Special thanks is also due to Mr. Wayne Sue Tin, an excellent teacher without whom this entire project would have been impossible. Another great teacher who lent a hand was Mr. Mark Edwards, who was always willing to help and discuss design issues.

Another extremely important person in the process of writing this thesis is Mr. John Sweeney, husband and best friend, who was source of help, encouragement and comfort at all times.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF APPENDICES

DEFINITIONS

RCBC. Reinforced concrete box culvert, an inverted U-shape type of culvert structure

Large RCBC. An RCBC that exceeds 1200 mm in span or 1200 mm in height and does not exceed 4200 mm in span and 4200 mm in height.

Three-sided box culvert. An RCBC that has two legs and a crown. If this structure requires a base, it is normally supplied separately or poured insitu.

Four-sided box culvert: An RCBC that has a box format, that is, two legs, a crown and a base all cast in as one structure.

SM1600. A representation of the W80, A160, M1600 and S1600 design loads.

ESO. Evolutionary Structural Optimisation; a topology optimisation method

BESO. Bi-directional Evolutionary Structural Optimisation; a topology optimisation method.

STANDARDS AND TECHNICAL SPECIFICATIONS

AS1597.2. Australian Standard: Precast reinforced concrete box culverts – Part 2: Large culverts

AS3600 Australian Standard: Concrete Structures

AS5100 Australian Standard: Bridge Design

AS5100.2 Australian Standard: Bridge Design – Part 2: Design Loads

MRTS24 Transport and Main Roads Specification: Manufacture of Precast Concrete Culverts

Technical Note 20a Transport and Main Roads Technical Note: Design Criteria for Large Box Culverts to MRTS24

CHAPTER 1 - INTRODUCTION

1.1 Background

Reinforced concrete box culverts are extensively used structural elements to convey flow of stormwater or sewerage. The typical box culvert produced in Australia has three sides and it is shaped like an inverted U. It is widely manufactured by precast concrete manufacturers, normally in steel moulds with fixed or variable sizes.

The Australian Standard that governs the production aspects of this product is currently AS1597.2-2013, which supersedes the 1996 version. Part 2 deals with large box culverts, those with span and height between 1.2 m and 4.2 m. This Standard gives preferred internal sizes for the box culverts (span and leg height), and lengths are normally 1.2 m or 2.4 m, with a few exceptions, depending on the manufacturer. Also, the Department of Transport and Main Roads Queensland (TMR) specifies a few criteria that are to be met should the large RCBC be installed under a main road in Queensland. The relevant document is the MRTS24 (Aug/11), with which the culverts in this study will also comply.

This study is concerned with finding the optimum topology for reinforced concrete large box culverts so that the final products are useful to the industry. To achieve this objective, the RCBCs will be compliant with AS1597.2-2013 and MRST24 (Aug/11) since without this compliance these culverts could not be sold or installed under main roads. In addition, the culverts will potentially be cheaper since they will utilise less material.

1.2 Research scope and objectives

This research has four main broad objectives:

1. Find the worst case among the applicable load combinations dictated by AS1597.2-2013

Firstly an analysis of all the possible load cases will be carried out to determine the worst case to be used in design. The RCBCs in this study will be assumed to be subjected to SM1600 loads, which is the standard traffic load for large box culverts according to Standards Australia (2013, p. 27). The SM1600 loads are a combination of the single wheel W80 load, the single axle A160 load and the moving tri-axle M1600 load according to AS1597.2-2013.

2. Design the standard RCBC complying with AS1597.2-2013 and MRTS24 (Aug/11)

   The design loads found in the previous analysis will then be utilised to design the RCBC utilising standard practices dictated by AS1597.2-2013 and MRTS24 (Aug/11). Finite element analysis using the software Strand7 will be employed to find design moment and shear capacity. The reinforcement will then be designed with the application of the concept of equivalent concrete compressive stress block for flexure analysis. The standard culvert will later be compared with the optimised culvert to evaluate its feasibility.

3. Optimise the RCBC using the SIMP method and finite element analysis

   The topology optimisation procedure will be implemented using finite element analysis based on the Solid Isotropic Material with Penalisation (SIMP) method. The analysis starts with a design domain with finite elements with relative material densities of 1, and in each iteration the topology optimisation equation is solved for each element until the objective function has been reached. In this case, the objective function will be to diminish strain energy and therefore to increase stiffness, with a minimum volume constraint. The SIMP method models material properties as the relative density of each finite element raised to a power, called the penalisation power, in order to diminish the occurrence of intermediate densities.

4. Analyse the feasibility of the optimum RCBC

   Once the optimum culvert has been found, a feasibility analysis will be carried out to evaluate the possible advantages of using this method to design and manufacture RCBCs over utilising standard methods. The cost of labour and materials, the time to produce the reinforcement cages, the extra time and cost to construct/prepare moulds, cast and install the units, among others, will be taken into consideration and a conclusion will be reached regarding feasibility of the optimum culvert.

This research will investigate 5 of the most commonly sold sizes of RCBC that are required to comply with Main Roads specifications as well as Australian Standards. The loads applied to the culvert will be SM1600 loads as specified by Standards Australia (2013), excluding heavy load platform loads (HLP) and railway loads. The desired outcome of this project is to find an optimum RCBC that is compliant with Australian Standards, useful to the industry, cheaper to manufacture and consequently possibly cheaper to the final customer and that is environmentally responsible because it will utilise less material.

CHAPTER 2 - LITERATURE REVIEW

2.1 Introduction

A review of literature for this research has identified a gap in the research for optimum box culverts that can be fabricated and utilised by the industry in Australia and this study endeavours to make a contribution to filling this gap.

There are not many FEA models to describe three-sided large box culverts, which is the type of RCBC commonly sold in Australia. The majority of papers describe procedures and results that apply to 4-sided culverts. In addition, there is not much research about topology optimisation of box culverts specifically. The published work mentions beams or multi-frames.

2.2 Optimisation

The word optimum comes from the Latin *optimus*, meaning 'best' or 'very good'. Optimisation is about generating the best possible design. However, there are different ways to define what is best in terms of design. Some may believe the cheapest product to be the best, while the design that uses the less amount of material and therefore provokes the lowest impact on the environment might be considered best by others.

During the design, manufacture and installation phases of construction of a structure, many parameters could be optimised. During design, one could aim for the shortest design time which would translate into savings for the company in terms of less hours required from the engineering team. It could also bring the company a competitive advantage if they are able to submit their design proposal before their competitors. This could mean the structure does not utilise the least amount of material and it may not have the most efficient size and shape but in that situation, it may be the only way the company is going to be hired to do the job.

As for manufacture, the most efficient reinforcement may be composed of various bar diameters and lengths. This normally means it would take longer for the reinforcement cage to be assembled since there is more measuring and cutting involved, incurring extra labour costs if compared with a cage composed of the same bar diameter of same or length at constant spacing.

In the installation phase, if the optimum structure has to be transported in a different position, for example, there could be higher installation costs including crane time and labour to rotate units. Another possibility is when the optimum structure is thinner that the standard structure and would not be able to be lifted the same way. The optimum structure could require spreader beams to be utilised, more rotations or changing of lifting anchors during installation, which also adds to crane hire and labour costs.

For these reasons, optimisation must be a trade-off between what is desirable, the optimum structure, and what is feasible, both structurally and commercially.

The literature contains numerous approaches devised to achieve various levels of optimisation. The most common ones are now discussed.

2.3 Topology optimisation

There is a myriad of approaches in literature to solve topology optimisation problems, i.e. the homogenization based approach, the Solid Isotropic Material with Penalization (SIMP) approach, evolutionary design methods and level set methods, to name a few.

2.3.1 Solid Isotropic Material with Penalization (SIMP)

The SIMP method was first described by Martin Bendsøe in 1989. (Rozvany 2001). This method models material properties as relative densities where a relative density of 1 indicates the solid material, a density of 0 models a void and a density between 0 and 1 means the material has voids at a microlevel. To ensure the material can be realized in practice as composites of the original material, Bendsøe and Sigmund (1999) have stated that p must satisfy the condition $p \geq \max\left\{\frac{2}{1-\nu}, \frac{4}{1+\nu}\right\}$ where ν is the Poisson ratio of the solid material.

One problem that can affect the SIMP results is mesh-dependence, which causes different solutions to be obtained depending on mesh sizes or discretization, instead of a more detailed solution of the same optimal structure (Sigmund & Petersson 1998). One way of preventing this from happening is to introduce a mesh-independence filtering scheme, which works by modifying the element sensitivities and is very simple to implement. (Sigmund 2001)

Sigmund (2001) distributed online a 99 line MATLAB code based on the SIMP method that solves the optimisation problem by applying Optimality Criteria (OC) methods. These are indirect methods developed in an attempt to diminish the number of design variables in the optimisation process (Hassani & Hinton 1998).

2.3.2 Homogenization based approach

This method uses a density of a composite with voids. When the density variable is 0, there is no material (void). When it is 1, there is material (solid). If the density is between 0 and 1 there is a porous component with voids at microlevel. The difference between the homogenization based approach and the SIMP method is that in the homogenization method, the material property of each finite element is obtained using the homogenization theory, and the optimal topology is achieved by solving a material distribution problem, while in the SIMP method the intermediate densities are penalised using the power-law approach, requiring no homogenization. One disadvantage of this method is that it may produce infinitesimal pores in the materials that impede construction. (Zhao, Long & Ma 2010).

2.3.3 Evolutionary approaches

A popular topology optimisation approach is the Evolutionary Structural Optimisation method. The basis of the ESO method is to remove inefficient material from the initial structure until a target condition is reached. The efficiency of the material is evaluated by the level of stress or strain energy in each element. However, ESO work published in the 1990s disregarded key aspects of topology optimisation i.e. existence of a solution, checker-board, mesh dependency and local optimum (Huang 2010).

To overcome these faults, a method called Bi-directional Evolutionary Structural Optimisation was developed by Huang and Xie (2007). This method allows elements to be added at the same time as they are removed and it also deals with the shortcomings mentioned above in the ESO method by utilising a mesh-independency filter and by including historical information of the sensitivity numbers of each element to improve their accuracy (Huang & Xie 2007).

2.3.4 Level-set method

Level-set methods were first introduced by Osher and Sethian to model moving boundaries (Van Dijk et al. 2013).

Shojaee and Mohammadian (2012) explain that this method depicts the transformation of an interface between two domains. It utilises a level-set function to describe the boundary as the zero level set, while nonzero level sets are used in the domain. While the optimization iterations are occurring, the level set surface may move causing the boundary to suffer considerable changes.

Wang, Wang and Guo (2004) utilised the level-set method in a boundary optimisation problem. The domain is represented by a level-set model embedded in a scalar function, governed by a Hamilton-Jacobi convection equation. This yielded a 3D structural optimisation technique which gives results comparable to other established optimisation techniques.

Yamada et al. (2010) proposed a new optimisation technique utilising the level set method and incorporating a fictitious interface energy (Chan-Hilliard energy) to overcome numerical instability problems such as mesh-dependency, checkerboard patterns and greyscales. Their results showed, through various numerical examples, minimal dependency on the finite element size or initial configurations.

2.4 Cost optimisation:

Sarma and Adeli (1998) present a review on different approaches used and cost savings achieved in different reinforced concrete structure optimisation papers from 1970 to 1996. These structures include beams, slabs, frames, plates and water tanks, among others, based on standard codes from the USA, Britain, Canada, India, Europe and Australia. No mention is made of box culverts in this literature review. The authors claim that optimising the weight of the structure will not necessarily produce the optimum design, since three parameters greatly influence the final cost of the concrete structure: concrete, steel and formwork. Therefore, the authors conclude that it is necessary to take a more general approach when considering cost optimisation and that this practice can result in significant savings.

Ignacio Martin has brilliantly stated in his discussion of the paper by Sarma and Adeli (1998): "An experienced builder can erect a safe structure, but only engineers can design economical safe structures" (Martín, Adeli & Sarma 1999). The discusser also states that defining cost optimisation is not an easy task and it should take into consideration parameters like function, availability of space, life cycle, construction time and marketability, among others.

Stanton and Javadi (2014) developed a finite-element based least cost optimisation Excel spreadsheet, ResOpt, that models optimum reservoirs using genetic algorithm as a basis for the optimisation process. The authors show how ResOpt produced cost savings of over 21% when utilised to model a 13Ml reservoir in Cornwall, UK.

Stanton and Javadi (2014) allege there to be a recent trend toward optimisation of structures that encompasses the life-cycle of a building, including the design phase, construction, maintenance and demolition.

2.5 Size optimisation

Zhu et al. (2012) demonstrated that finite element analysis was successfully utilised to optimise the four-sided box culvert structures built under a highway in the Tuanbo Reservoir area, in Tianjin, China. The authors analysed dozens of combination of sidewall thicknesses and baseplate thicknesses in ABAQUS under vehicle loads as per the General Code for Design of Highway Bridges and Culverts (China Ministry of Transport 2004). Using FEM analysis to simulate stresses and deformations, the authors obtained the optimised culvert. In this case, the optimum culvert was the one that met the stress and deflection requirements of current bridge specifications in China and had minimum weight. The reduced self-weight also resulted in decreased soil bearing capacity requirements. The final structure was cast insitu and was composed of two 22 metres long sections with a width of 24.2 metres.

2.6 Shape optimisation

Rath, Ahlawat and Ramaswamy (1999) developed a design procedure that optimises the shape of flexural members' cross sections made of anisotropic materials like reinforced concrete. The aim is to minimise total cost, which in this case is made up of material, manufacture and placement costs. The authors assert that if there is more material in high stress zones, the use of materials will be more efficient and will result in savings. The procedure is exemplified in the design of three types of beams: simply supported, cantilever and 2-span continuous beams. Finite element modelling, natural velocity field method and genetic algorithms were utilised.

2.7 Reinforcement optimisation

Aschheim, Hernandez-Montes and Gil-Martin (2008) propose simpler approach to design optimum reinforced concrete beams, walls and columns that does not require tables or interaction charts. The authors show the design procedure of a reinforced concrete section under an axial force and a moment, using nonlinear conjugate gradient search technique. A proposed single model can then be used for beams, walls and columns. The authors' approach can also be integrated in widely available spreadsheet programs. By finding the optimum design, which is this case means the design with minimum reinforcement and minimum concrete, the authors claim to improve the sustainability of reinforced concrete construction.

Gil-Martin et al. (2011) presented and proved a theorem they called TORS – theorem of optimal section reinforcement. This theorem establishes which cases of bottom and top reinforcement will result in minimum reinforcement, using ACI-318-08 assumptions. The theorem states that the minimum total reinforcement area occurs for one of the four following cases:

1. The bottom reinforcement area and/or the top reinforcement area is zero

2. The strain at the bottom reinforcement (ε_S) is equal to or slightly greater than the yield strain of the reinforcement ($-\varepsilon_Y$)

3. The strain at the top reinforcement and bottom reinforcement are equal to the maximum concrete strain of 0.003 ($\varepsilon = \varepsilon_S = \varepsilon'_S = \varepsilon_{C.max} = 0.003$)

4. The strain at the top reinforcement (ε'_S) is equal to the yield strain of the reinforcement ($-\varepsilon_Y$)

The authors also state that there is an infinite number of admissible reinforcement solutions for each problem, but their proposed theorem enables a quicker solution using optimum quantity of reinforcement without the need for reinforcement sizing diagrams by evaluating the four cases above. The study indicated that the optimum solution, the one that uses a minimum amount of reinforcement, is significantly different than the typical symmetric reinforcement solution shown in standards and textbooks.

2.8 Optimisation and Constructability

Guest and Moen (2010) employed topology optimisation methods to truss analysis and developed an optimisation routine aimed at reducing crack widths and enhancing member performance compared to traditional strut and tie models. It allows engineers to visually examine designs and enables them to identify the stiffest truss which describes the flow of forces in a general concrete member with general loading and support conditions.

However, if constructability is not taken into consideration there is a great chance the resulting design will not be achievable in practice. To correct that, Zhu et al. (2014) propose that constructability measures should be inserted into free-form topology optimisation as constraints and/or objective functions. This will ensure that constructability, which according to the authors is typically the primary governing cost in building a structure, is taken into consideration and that the optimisation process yields results that can be applied in practice.

Guest et al. (2012) also defend that although topology optimisation can yield valid design ideas, it is often prohibitively difficult to build these optimum structures. To help mitigate the negative effects of difficult constructability, the authors developed algorithms to: influence the constructability of systems and manufacturability of components; utilise nonlinear material models to optimise design and improve optimisation by considering fabrication or construction errors or damage.

One way to improve constructability is to restrict the geometric design space. (Guest et al. 2012) This has been done by Stromberg et al. (2011) using pattern gradation and repetition. This means that restrictions are placed regarding number and variable size of repeating patterns along any direction on the design domain, which results in enhanced constructability.

Guest, Prévost and Belytschko (2004) also managed to enhance constructability by restricting the diameter of the designed members. The authors used nodal volume fractions as a design variable, making element volume fractions a function of the nodal volume fractions.

It is also possible to regulate the maximum length scale of members. Guest (2009) showed that by searching the design domain and applying local constraints that will impede the development of features that are larger than a required maximum, it is possible to improve constructability.

2.9 Using finite element analysis (FEA) to model RCBC behaviour

There are mainly two types of box culverts: three-sided, which do not have a base slab, and four-sided, which have a box format. In Australia, the tree-sided culvert is the type used in the great majority of construction projects.

Figure 2-1 - Four-sided box culvert (Foley Products 2014)


Figure 2-2 - Three-sided box culvert (Rocla 2014)

Many of the studies available that use FEA to model RCBC behaviour utilise the four-sided culvert, since this type is vastly used overseas. Some of these studies are now presented.

Awwad et al. (2000) performed a three-dimensional finite element analysis of four-sided large reinforced concrete box culverts using the software SAP 2000. The culverts analysed had spans in excess of 3.6 metres fill heights between 0 and 3 metres. Live loads such as AASHTO H20 truck were applied, as well as overburden pressure, lateral pressure and bearing pressure.

McGrath, Liepins and Beaver (2005) performed three-dimensional analyses on four-sided reinforced concrete box culverts with depths of fill up to 0.600m subjected to live loads according to the American Association of State Highway and Transportation Officials (AASHTO) load and resistance factor design (LRFD) introduced in 1994.

A PhD thesis published in the University of Texas (Garg 2006) simulated experimental tests done in four-sided RCBCs using the finite element modelling software ABAQUS. The author used three-dimensional shell and solid elements as well as welded wire fabrics to reproduce the behaviour of the RCBC and its reinforcement in order to draw conclusions regarding the appropriateness of the ASSHTO 2005 shear provisions across the culvert joint. It was also concluded that the results obtained by the 3D FEM analysis in relation to deflections corresponded with the experimental results.

Ahmed and Amanat (2008) argue that a two-dimensional analysis of four-sided reinforced concrete box culverts, which is the basis of the design procedure in Canada, is unable to realistically model the interaction between the buried box culvert and the soil above it in deeply buried RCBCs. They affirm that a detailed three-dimensional finite element analysis is required to enable an evaluation of the stresses developed in the flow direction and consequently to realistically model the box culvert interrelationship with the soil.

However, Awwad et al. (2008) contend that for four-sided box culverts with spans of 3.6m, a plane frame analysis outputs less conservative moment and deflection results than a three-dimensional finite element analysis. The authors performed a parametric study on three sizes of culverts with spans of 3.6m, 5.4m and 7.2m. For fill depths under 0.9m, the wheel loading was found to be dominant. However, for fill depths between 2.1m and 3m, the position of the wheel along the midspan of the culvert slab was found not to yield considerably different results with respect to earth loading. As for fill depths over 3.0m, it was found that these results did not differ at all.

Kang et al. (2008) used the software programmes CANDE (Culvert ANalysis and DEsign), ABAQUS and MSC/NASTRAN to investigate the effects of frictional forces on the sidewalls of four-sided RCBCs.

Garg and Abolmaali (2009) used ABAQUS to simulate four-sided RCBC behaviour and compared them with previously done experimental tests. The finite element models used showed cracking propagation patterns were very similar to those found in the experimental results. The box culverts used in this study had the standard sizes according to ASTM-C-1433-04.

Chen, Zheng and Han (2010) used the commercially available geotechnical finite element software PLAXIS to investigate factors that influence vertical earth pressures onto four-sided culverts, including height of fill and dimensions of the culvert. The reinforced concrete culverts were modelled as an elastic material and the study concluded that the Chinese General Code for Design of Highway Bridges and Culverts provide conservative methods to estimate earth pressures on culverts.

PLAXIS was also used by Kim et al. (2011) to perform a finite element analysis of a four-sided 1.8m x 1.8m reinforced concrete box culvert with an inlet opening at the top. This culvert presented severe cracking and was about to collapse in Georgia, USA. The results of the analysis were used to provide repair alternatives to prevent complete failure.

Das (2013) utilised 3D-FEA to perform a refined load rating procedure on four four-sided box culverts, three of which were built before 1940 while the fourth was built in 1985. The author concluded that the results between the conventional rating analysis, based on ASSHTO's Allowable Stress and Load Factor rating method, and his refined 3D-FEA could vary by more than 250%, depending on the physical conditions of the culverts, field measurements and load test data. Das (2013) states that the improvement brought by the 3D-FEA method is due to appropriate use of a few factors including realistic live load distribution obtained from 3D-FEA.

In contrast with the great amount of studies about four-sided box culverts, there appears to be very few studies on three-sided box culverts. Frederick and Tarhini (2000) pointed out that the American Society for Testing and Materials (ASTM) did not discuss three-sided culverts. To fill this gap, the authors used three-dimensional finite element analysis to analyse and design three-sided box culverts with spans between 4m and 11m with less than 0.6m of fill and subjected to live load, impact load, dead load and lateral earth pressure. Frederick and Tarhini (2000) concluded that the requirements established by ASSHTO and ASTM were met when analysing these structures using plane frame analysis or 3D FEA, with the latter having the advantage that is gives values for the transverse bending moments and shear forces, which in the case of this study were very low.

FEA is also applicable to various other structures. Regarding pipes, for example, Kitane and McGrath (2006) stated that although two dimensional analysis was suitable for situations when the pipe culverts are deeply buried, it may lead to conservative designs should the culvert be buried closer to the surface and subjected to live loads.

2.10 Conclusions

The literature revealed a multitude of possible uses of FEA to model structural behaviour. There is, however, a lack of research specifically on three-sided reinforced concrete box culverts, which is the most common type of box culvert found in Australia. By applying FEA to large box culverts in search for the optimised structure, there will be great gain to the industry, to the end customer and the environment.

This research will utilise the SIMP method due to its mathematical simplicity since it does not require derivations including higher mathematics; its computational efficiency due to the utilization of a single free variable per finite element; and the fact that it does not require homogenization, only adjustment of a suitable penalization factor.

CHAPTER 3 - METHODOLOGY

3.1 Introduction

Various topology optimisation methods have been extensively studied. However, the most popular manner to introduce the concept of topology into structural analysis is via the Solid Isotropic Material with Penalisation (SIMP) method. (Bruns 2005). This method assigns a value to the relative density of each finite element in the domain, and penalizes the intermediate values between 0 (void) and 1 (solid material) more heavily in order to generate solid-void structural designs. Some advantages of this method are that it is computationally efficient, it can be used for any combination of design constraints and it is conceptually simple without requiring derivations involving higher mathematics (Rozvany 2001). Due to these characteristics, the SIMP method will be utilised in this project.

RCBCs which need to comply with MRTS24 comprise of approximately 40% of the large box culvert sales where Roome (2014) currently works. The other 60% of culverts are for subdivision works, but they generate less sales volume per each project. This is one of the reasons why this project focuses on Main Roads culverts. Also, the most common box culvert sizes sold according to Roome (2014) are between 1.8 m span by 1.5m leg (1815 RCBC) to 2.4 m span by 1.8 m leg (2418 RCBC). That interval comprises a total of 5 box culvert sizes out of the 24 possible sizes. Roome (2014) estimates these sizes make up between 60% of all large box culvert sales. Due to the commercial significance of these Main Roads RCBC sizes, they will each be investigated in detail.

To find the optimum RCBC, four main steps will be required. Firstly, the SM1600 loads will be analysed and the worst case for each part of the culvert will be found and taken as design load. Then, the standard RCBC will be designed according to AS1597.2-2013 and MRTS24 (Aug/11) to be later compared with the optimum RCBC. The next step will be to find the optimum topology for the RCBC under the design loads utilising the SIMP method and finite element analysis. Lastly, a feasibility analysis will be carried out to outline the benefits and drawbacks of utilising this optimisation procedure in the industry. These four steps are discussed in the next sections.

3.2 Analysis of SM1600 loads

The live loads for road bridge design in AS5100.2 (Bridge Design code) are referred to as the SM1600, which are made up of the W80, A160, M1600, and S1600 and M1600 design loads. (Standards Australia 2013, p. 58). These loads represent road traffic design loads for main and secondary roads, which are commonly specified in the industry. According to AS1597.2 (Large RCBC code), large reinforced concrete box culverts are to be designed for W80, A160 and M1600 as per AS5100.2 but excluding the uniformly distributed load component from the M1600 load (Standards Australia 2013, p. 27). Heavy load platform loads (HLP320 and HLP400) and railway loads (300LA) are not included in this study.

3.2.1 W80 load

The W80 load represents an individual heavy wheel load that uniformly imposes 80kN distributed over a contact area of 400 mm x 500 mm for the strength limit state and 200 mm x 500 mm for the serviceability limit state (Standards Australia 2013, pp. 28-31) (Standards Australia 2004a, p. 12).

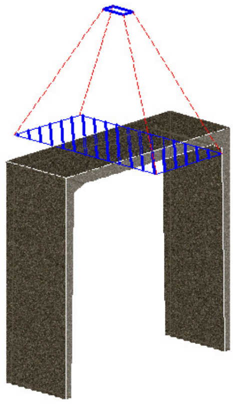

Figure 3-1 - W80 wheel load

Figure 3-1 shows the truncated prism representation for a W80 wheel load distributed through fill. The top blue rectangle represents the wheel contact area. The dashed red lines represent the truncated prism and the bottom hatched rectangles represent the area over which the pressure is distributed on top of the RCBC.

3.2.2 A160 load

The A160 load represents an individual heavy axle load that uniformly imposes 160kN distributed over a contact area of 400 mm x 500 mm for the strength limit state and 200 mm x 500 mm for the serviceability limit state The standard design lane size is 3200 mm and the distance between the two wheels in the axle is 2000 mm. (Standards Australia 2013, pp. 28-31) (Standards Australia 2004a, pp. 12-3).

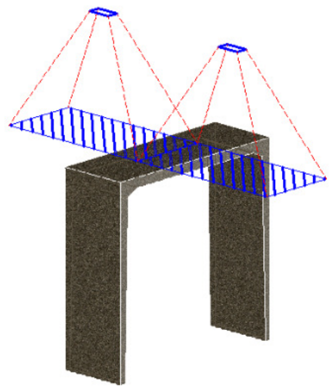

Figure 3-2 - A160 axle load

Figure 3-2 shows the truncated prism representation for an A160 axle load distributed through fill.

3.2.3 M1600 load

The M1600 load models two heavy vehicles in the same lane together with an accompanying stream of general traffic (Standards Australia 2007). Each heavy vehicle of M1600 has two tri-axles, one of which is represented in Figure 3-3, with the red lines representing the truncated prism load distribution through fill. Each axle is 1.25m from the other.

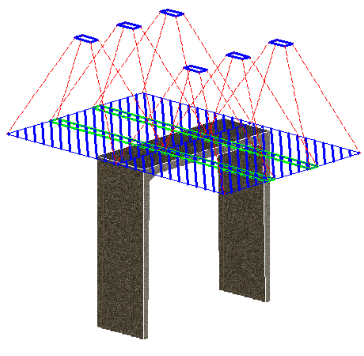

Figure 3-3 - M1600 tri-axle load

Figure 3-4 shows the M1600 moving traffic loads, with all dimensions in mm.

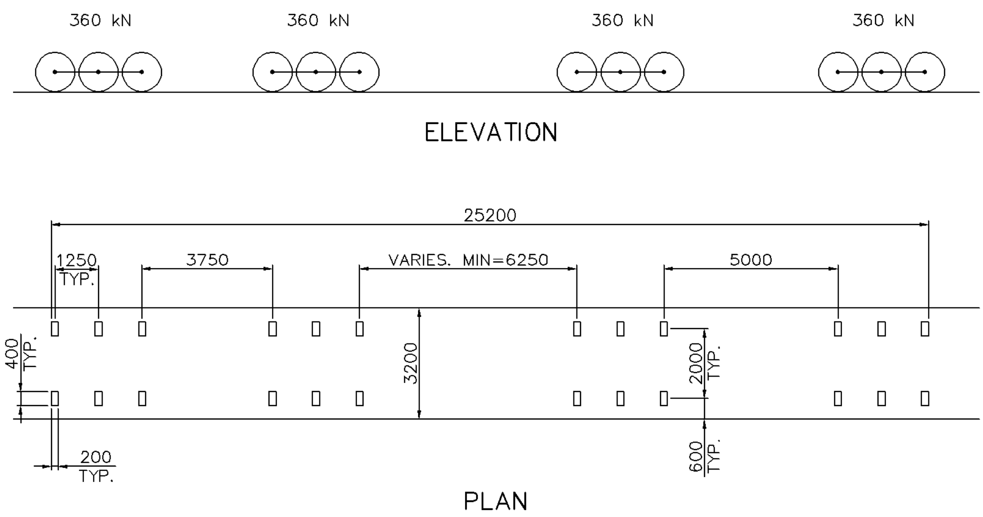

Figure 3-4 - M1600 according to AS1597.2

According to AS1597.2-2013, the minimum roadway load class to be considered is Class 2-A, which means the amount of fill considered is to be from 0m to 2m of fill inclusive. (Standards Australia 2013, p. 13). The road traffic loads are to be applied for the entire range of fill from 0m to 2m inclusive (Standards Australia 2013, p. 27). However, AS1597.2 also dictates the minimum fill directly above the top of the culvert (overlay) is to be at least 150 mm (Standards Australia 2013, p. 9). To be conservative, the calculations in this project will go from 0.1 m to 2.0 m of fill.

The distribution of loads through fill is calculated by using a truncated prism type approximation, in accordance to AS1597.2-2013 Clause 3.3.5.5.2 (Standards Australia 2013, p. 58). Graphic examples of the truncated prism can be seen in Figure 3-1, Figure 3-2 and Figure 3-3.

For the serviceability limit state, the W80 single wheel load pressure area is given by $A = L_1 L_2 = (b + 1.45H)(a + 1.45H)$ where

a=0.2 m

b=0.5 m

H = height of fill over RCBC in metres

It can be seen in Figure 3-3 that for the tri-axle case, the loads will overlap (shown in green). In this case, AS1597.2 stipulates the distribution area will still be even but will be given by $A = L_1 L_2 = (\mathrm{G} + b + 1.45H)(\mathrm{J} + a + 1.45H)$ where all parameters remain the same and

G=distance between wheels in metres

J=distance between axles in metres

For the ultimate limit state, the W80 single wheel load pressure area is given by $A = L_1 L_2 = (b + 1.15H)(a + 1.15H)$ where

a=0.4 m for W80 and A160 and a=0.3 for M1600

b=0.5 m

H = height of fill over RCBC in metres

Where the loads overlap, the distribution area will still be even but will be given by $A = L_1 L_2 = (\mathrm{G} + b + 1.15H)(\mathrm{J} + a + 1.15H)$ where all parameters remain the same and

G=distance between wheels in metres

J=distance between axles in metres

When designing the culvert according to AS1597.2-2013, the maximum superimposed load case should be used as the design basis. (Standards Australia 2013, p. 57). The critical load combination will be determined according to AS1597.2-2013 Clause 3.4, which dictates culverts are to be designed to resist loads imposed onto them during intermediate and final stages of construction. (Standards Australia 2013, p. 32). The culverts will be subjected to both construction loads and in-service loads and both in the horizontal and the vertical directions.

The vertical loads applicable to this study are:

- W_{DC}: self-weight
- W_{FV}: vertical earth pressure due to fill
- W_{CV}: construction live load induced vertical earth pressure
- W_{LV}: roadway live load induced vertical earth pressure

The horizontal loads applicable to this study are:

- W_{FH}: horizontal earth pressure due to fill
- W_{AH}: horizontal pressure due to compaction
- W_{CH}: construction live load induced horizontal earth pressure
- W_{LH}: roadway live load induced horizontal earth pressure

As mentioned earlier, heavy load platform loads (HLP320 and HLP400) and railway loads (300LA) are not included in this study.

3.2.4 Load factors

The load factors dictated by AS1597.2-2013 for stability and strength limit states are shown in Table 3-1 and Table 3-2.

Table 3-1 - Stability and strength limit state load factors for vertical loads

Load		Load Factor	Alternative Load Factor
W_{DC}	self-weight	1	-
W_{FV}	vertical earth pressure due to fill	1.4	0.9
W_{CV}	construction live load induced vertical earth pressure	1.5	0
W_{LV}	roadway live load induced vertical earth pressure	1.8	0

Table 3-2 - Stability and strength limit state load factors for horizontal loads

Load		Symmetric loading		Asymmetric loading	
		Load Factor	Alternative Load Factor	On one side	On opposite side
W_{FH}	horizontal earth pressure due to fill	0.7	1.4	1.4	0.7
W_{AH}	horizontal pressure due to compaction	0.7	1.4	1.4	0.7
W_{CH}	construction live load induced horizontal earth pressure	1.5	0	-	-
W_{LH}	roadway live load induced horizontal earth pressure	1.8	0	-	-

Some loads have two possible load factors: one higher than unity and one lower than unity. When a change in the situation being analysed (like an increase in the load) decreases safety, a load factor greater than unity is used and when it increases safety (like a decrease in the load), the load factor is smaller than one.

3.2.5 Self-weight W_{DC}

The vertical loads to be considered will obviously always include the culvert self-weight, which has a load factor of 1. According to section G2 of Standards Australia (2013), in the absence of more specific material information, the self-weight should be calculated assuming a reinforced concrete density of 2650kg/m^3 and a gravity force per unit volume of 26.0kN/m^3. The volume of each culvert was calculated in AutoCAD and the mass was found utilising the above density.

3.2.6 Vertical earth pressure due to fill W_{FV}

In embankment installation conditions, the side zone material shall extend out horizontally for a width equal to one-third of the height of the culvert or a minimum width of 300 mm, whichever is greater (Standards Australia 2013, p.

42). The vertical earth pressure due to fill (W_{FV}) for embankment installation is measured in kPa and is given in AS1597.2-2013 by $W_{FV} = (1 + 0.2\frac{H}{B_c})\gamma H$

where H is the height of fill over the culvert, from 0.0 m to 2.0 m

Bc is the is the overall outside width of the culvert in metres

γ is the gravity force per unit volume of the fill material, assumed 20kN/m^3

There are two possible load factors to be used. In terms of vertical earth pressure due to fill, a situation with a small height of fill would act beneficially to dissipating the live and construction loads on top of the culvert, therefore the most appropriate load factor would be 0.9. However, as fill depths increase, their beneficial action to dissipating loads on top of the culvert is countered by the pressure the greater amount of fill actually puts on top of the culvert. In this situation, the most appropriate load factor is 1.4.

3.2.7 Construction live load induced vertical earth pressure W_{CV}

The induced vertical earth pressure caused by the construction live loads is to be taken into consideration during the intermediate stages of construction and it is to be applied at 0.4m of fill or at the final fill height, if less than 0.4m. (Standards Australia 2013, p. 28) Depending on the depth of fill, the critical case will vary between the W80, A160 and M1600 load cases. That is because the shallower the depths, the less room there is for load distributions from multiple axes based on the truncated prism to fully overlap.

3.2.8 Vertical loads due to road traffic loadings W_{LV}

The effect of roadway live load induced vertical earth pressure is to be calculated by dividing the sum of the wheel loads applying pressure to the culverts by the area of application, based on the truncated prism method. A dynamic load allowance (DLA) is also applied and it varies linearly from 0.4 at

0m of fill to 0.1 at 2.0m of fill. (Standards Australia 2013, p. 30). This translates to $W_{LV} = (1 + DLA) \cdot \frac{\sum P}{A}$

where DLA is dynamic load allowance from 0.4 to 0.1

$\sum P$ is the is sum of the individual wheel loads applying pressure to the culvert in kN

A is the area of the truncated prism base in m^2

There are five options for the critical case of live loads due to traffic:

- W80 load on single lane
- A160 load on single lane
- M1600 load on single lane
- A160 load on dual lane
- M1600 load on dual lane

The case with W80 wheel load on a dual lane is not considered because it produces localized effects and therefore is not appropriate for dual lane (Standards Australia 2013, p. 58).

AS1597.2 brings the critical cases for SM1600 in table G3, which are used in this study (Standards Australia 2013, p. 62). For fill depths up to 1.2 m the critical case is the single lane W80. From fill depths of 1.3 m to 2.0 m, the critical case is the dual lane A160.

3.2.9 Horizontal loads due to fill and compaction W_{FH} and W_{AH}

When it comes to horizontal loads due to fill and compaction, there are two situations to be analysed: when the load on both sides of the culvert is the same (symmetric loading) and when they differ one from the other (asymmetric loading).

In the symmetric case, there are two possible load factors, one greater and one lower than unity. This is due to the fact that higher horizontal forces generated by fill and compaction act favourably to strengthening the culvert crown in bending, since they counter the moment generated at the edge of the crown. In this situation, the most appropriate load factor would be 0.7. In contrast, it is possible that the fill will not act favourably to strengthening the culvert in bending, for example in case of poor compaction, when the culvert crown bends more freely without as much restraint from fill and compaction. Then, the appropriate load factor would be 1.4.

Regarding asymmetric loading, it is also required to check for the worst combination. A check is required with the lower load factor of 0.7 on one side of the culvert and the higher load factor of 1.4 on the other side and also with these reversed.

3.3 Designing the standard RCBC

The culverts were designed utilising linear structural analysis combined with ultimate strength theory, as dictated in the Concrete Structures code AS3600 (Standards Australia 2009, p. 28). It is known that in practice when reinforced concrete structures are subjected to loads they do not behave linearly. However, the Australian Standard Codes followed in this study not only permit the use of linear analysis but also impose the use of safety coefficients at several stages of design. Since the objective of this study is not to analyse structural failure, the use of linear analysis and safety coefficients is deemed sufficiently accurate.

According to the Main Roads Standard Specification MRTS 24 (06/09) and AS1597.2-2013, the culverts are to be designed as portal frames and the supports shall be modelled as pins at the base (Transport and Main Roads 2010a, p. 3) (Standards Australia 2013, p. 24). Also, sidesway does not need to be taken into consideration if the culvert is installed according to AS1597.2-2013 Section 5. (Transport and Main Roads 2010b) (Standards Australia 2013, p. 34). It is therefore assumed in this study that culverts are installed according to AS1597.2-2013 Section 5.

AS1597.2 describes the preferred internal dimensions of large RCBCs, which are normally observed by manufacturers. As the intention of this project is to generate results and conclusions that can be applied in practice, the culverts were modelled to have the same internal dimensions as described in AS1597.2.

Table 3-3 - Preferred RCBC internal dimensions

Size class mm	Nominal span mm	Nominal height mm
1500 x 900	1500	900
1500 x 1200	1500	1200
1500 x 1500	1500	1500
1800 x 1200	1800	1200
1800 x 1500	1800	1500
1800 x 1800	1800	1800
2400 x 1200	2400	1200
2400 x 1500	2400	1500
2400 x 1800	2400	1800
2400 x 2400	2400	2400
3000 x 1200	3000	1200
3000 x 1800	3000	1800
3000 x 2400	3000	2400
3000 x 3000	3000	3000
3600 x 1200	3600	1200
3600 x 1800	3600	1800
3600 x 2400	3600	2400
3600 x 3000	3600	3000
3600 x 3600	3600	3600
4200 x 1800	4200	1800
4200 x 2400	4200	2400
4200 x 3000	4200	3000
4200 x 3600	4200	3600
4200 x 4200	4200	4200

The general design requirements described by AS1597.2 are that culverts are to be designed to satisfy stability, strength, serviceability and durability limit states (Standards Australia 2013, p. 24).

3.3.1 Materials

The concrete utilised has an assumed Poisson's ratio of 0.2, in accordance with AS3600-2009 Clause 3.1.5. Also, clause 3.1.3 of the same standard stipulates the density of normal-weight concrete is to be taken as 2400kg/m^3, unless specific laboratory results are available. (Loo 2010, pp. 13-4) The concrete is assumed to have characteristic strength of 50MPa as per MRTS24 clause 10.7 and therefore have a Young's Modulus of 34800MPa.

The steel reinforcement is assumed class N deformed bar (designation D500N) with yield stress of 500 MPa. The elastic modulus of the steel reinforcement is assumed to be 200 GPa for both tension and compression (Foster 2010, p. 532).

3.3.2 Durability Design

MRTS24 states that large box culverts shall be designed for a minimum exposure classification of B2 in accordance with AS 5100. This standard caters for reinforced concrete structures and members with a design life of 100 years. The exposure classification B2 is appropriate for surfaces of members in above-ground exterior environments in coastal areas in any climatic zones. This means the culverts can be up to 1km from the coastline but not in tidal or splash zones. Members can also be permanently submerged in sea water (Standards Australia 2004b, p. 29). Table 4.5 from AS5100.5 shows the requirement for at least 25 MPa compressive strength at the completion of accelerated curing, such as steam curing, which is commonly used in the precast industry. Also, the minimum strength of concrete to be utilised is to be 40 MPa. This means the assumed 50 MPa concrete in this study complies as long as the product achieves at least 25 MPa during accelerated curing.

Abrasion also needs to be taken into consideration since the culvert is being designed for 0.0 m of fill. For medium or heavy pneumatic-tyred traffic, the minimum compressive strength required by AS5100.5 is 32 MPa, and for non-pneumatic-tyred traffic the minimum is 40 MPa. Again, the assumption made in this study is compliant.

The cover to reinforcing steel must be suitable for both the placement of concrete and for the protection of reinforcement against corrosion. For concrete placement, the cover shall not be less than the maximum between 1.5 times the maximum nominal size of the aggregate and the diameter of the reinforcing bar. In this study, it is assumed that the aggregate nominal size is 20 mm and the maximum bar diameter utilised in the culvert reinforcement is N28 (28 mm diameter), the cover should not be less than 30 mm (1.5 x 20 mm = 30 mm).

Because it is common in the precast industry to utilise rigid formwork such as rigid steel forms and intense compaction obtained with vibrating tables or self-compacting, super workable concrete, the nominal cover for 50 MPa concrete subject to B2 exposure classification is 35 mm (Standards Australia 2004b, p. 34), with a tolerance of -5, +10 mm. This means the minimum cover to reinforcement has to be 30 mm and the maximum cover has to be 45 mm, which is in accordance with the stated conditions for cover for concrete placement.

In summary, for the culvert to comply with durability requirements, it needs to:

- Have cover to reinforcement between 30 mm and 45 mm
- Utilise rigid formwork and intense compaction
- Have aggregate nominal size of no more than 20 mm
- Utilise reinforcement bars of the class D500N with a maximum diameter of 28 mm

- Be installed in conditions suitable for B2 exposure classification i.e. not in tidal or splash zones

3.3.3 Stability and Strength Design

According to AS1597.2 clause 3.6.1, design for strength shall be in accordance with AS3600, which dictates the allowable strength checks and methods of structural analysis. In this study, a strength check procedure for linear elastic methods of analysis and ultimate strength theory will be utilised. It is required that the design capacity of the cross section being considered is greater than the design action effects. When that concept is applied to a cross section of the crown or leg or the box culvert in bending, it yields $\phi M_u \geq M^*$ where

ϕ is the capacity reduction factor
M_u is the moment capacity of the section
M^* is the design ultimate moment

The design for stability shall comply with AS1597.2-2013 clause 3.4, which dictates the load combinations to be applied to culverts, as explained in Section 3.2.

3.3.4 Serviceability Design

According to AS1597.2 clause 3.6.2, design for durability shall be in accordance with AS5100 or AS3600. In AS3600, the key aspects of serviceability design are concerned with deflections and cracking of concrete (Foster 2010, p. 89).

Deflection was considered by using the simplified calculation for slab deflection described in AS3600-2009 section 9.3.3. A prismatic beam of unit width was the equivalent structure utilised. According to AS3600-2009 section 2.3.2, the deflection limitation on the crown $\left(\dfrac{\Delta}{L_{ef}}\right)$, which is subject to vehicular traffic, is to be less than 1/800. For the legs, the lateral deflection shall not exceed 1/500 of the leg height.

Shrinkage and temperature effects play an important role in concrete cracking and to control these effects distribution reinforcement must be provided in box culvert the crown and legs with a maximum bar spacing of 300 mm and a minimum area of 150 mm^2/m measured in the direction of the main flexural reinforcement (Standards Australia 2013, p. 35).

3.3.5 Non-optimised Culverts

To design the non-optimised culverts, the moment and shear capacity are determined using the finite element analysis software Strand7. The boundary conditions are introduced by restraining the nodes at the bottom of the culvert to model pins. In Strand7, the nodes have three translational and three rotational degrees of freedom (Strand7 Pty. Ltd. 2010). By fixing all but the rotational degree of freedom in the Z direction, which is the direction of the length of the culvert of 2.4 m, the boundary conditions are implemented by modelling pins as required by Transport and Main Roads (2010a, p. 3) and Standards Australia (2013, p. 24).

The crown and legs of the RCBC are modelled in Strand7 as beams and the vertical and horizontal loads are applied to them as distributed loads based on the critical loads determined in Section 3.2. The linear static analysis then yields the bending moment and shear force diagrams.

3.3.6 Reinforcement

The design process used to design the RCBC is iterative. Firstly an initial assumption is made regarding the thickness of the crown and leg of the structure. Then, the reinforcement is determined according to the procedures in AS3600 (AS1597.2 clause 3.5.1), with the number and diameter of required bars found iteratively with the Matlab program `flexanalysis.m` (see Appendix B). If the section does not have enough capacity, it is thickened and the process starts again.

The concept of equivalent concrete compressive stress block is utilised for flexure analysis by the Matlab program `flexanalysis.m`, described by two parameters:

Equation 3-1

$$\gamma = 1.05 - 0.007 f'_c \text{ within the limits of } 0.67 \le \gamma \le 0.85$$

Equation 3-2

$$\alpha_2 = 1.0 - 0.003 f'_c \text{ within the limits of } 0.67 \le \alpha_2 \le 0.85$$

These parameters dictated by AS3600-2009 ensure the total volume of the stress block is the same as the total volume of the equivalent stress block and that the centroid of the two blocks is also at the same height, as shown in Figure 3-5.

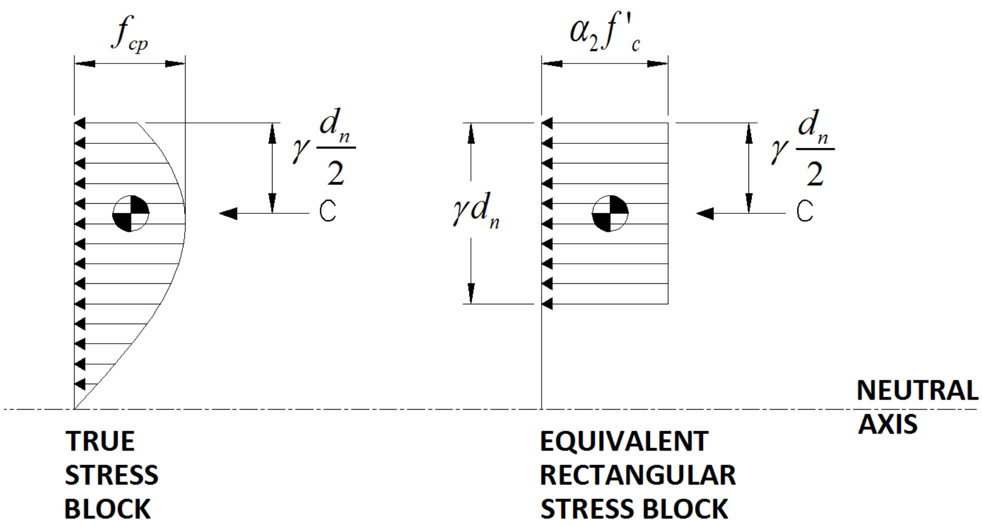

Figure 3-5 - Compressive stress block

The value for the extreme fibre concrete strain is adopted in AS3600-2009 as $\varepsilon_{cu} = 0.003$ and $\alpha_2 = 0.85$ for $f'_c \le 50MPa$.

Therefore, the forces calculated in the Matlab program `flexanalysis.m` are derived from the conditions at M_u (ultimate bending capacity) as shown in Figure 3-6.

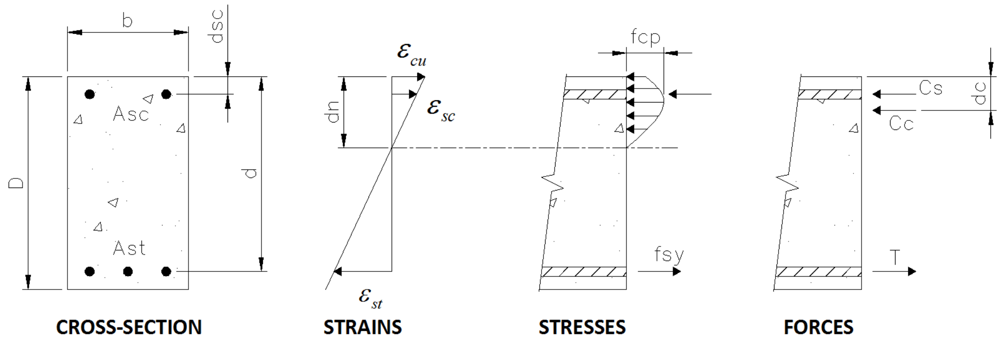

Figure 3-6 - Ultimate bending capacity conditions

These forces are:

Equation 3-3 - Forces at Mu

$$T = \sigma_s A_{st}$$

$$C_c = \gamma d_n b \alpha_2 f'_c$$

$$C_s = E_s \varepsilon_{sc} A_{sc}$$

The Matlab program `flexanalysis.m` (see Appendix B) then calculates d_n so that $C_c+C_s=T$, which is a condition for equilibrium. It then checks that $k_u = \frac{d_n}{d} < 0.36$ to ensure the section is under-reinforced and calculates $M_u = C_c(d-d_c) + C_s(d-d_{sc})$ and ϕM_u, with $\phi = 0.8$ for bending.

All details of reinforcement such as spacing, extensions and termination of reinforcement shall comply with AS3600-2009.

The development length for deformed bars in tension utilised in this study is the basic one described in AS3600-2009 Section 13:

$$L_{sy.tb} = \frac{0.5 k_1 k_3 f_{sy} d_b}{k_2 \sqrt{f'_c}} \geq 29 k_1 d_b$$

where

k_1= 1.3 for a horizontal bar with more than 300 mm of concrete cast below the bar or

= 1.0 otherwise

k_2= (132-d_b)/100 and

k3= 1.0 − 0.15(c_d − d_b) / d_b (within the limits 0.7 ≤ k_3 ≤ 1.0); where

cd = minimum between the distance between parallel bars and the cover to the bar from the tension face

In addition, according to AS1597.2 section 3.7.2, the minimum flexural reinforcement shall not be less than 0.002Ag in span direction, where Ag is the gross concrete cross-sectional area.

A shear check is also carried out to check if shear reinforcement is required and for shear design of RCBCs, Clause 8.2.7 of AS3600-2009 is normally applicable (Standards Australia 2013, p. 35). It dictates that the ultimate shear strength (V_{uc}) excluding the contribution of shear reinforcement is given by

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_0 f_{cv} \left(\frac{A_{st}}{b_v d_0} \right)^{\frac{1}{3}}$$

where

β_1= 1.1(1.6-d_0/1000)≥**0.8**

β_2= 1 for pure bending

β_3= 1

f_{cv}= $f'_c{}^{(1/3)}$≤ 4 MPa

A_{st} = cross-sectional area of longitudinal reinforcement provided in the tensile zone and fully anchored at the cross-section under consideration

d_0 = the distance from the extreme compressive fibre to the centroid of the most tensile reinforcement

b_v = effective width of the web

According to AS3600-2009, there are three possible cases in relation to shear reinforcement:

1. clause 8.2.5 (a): if $V^* \leq 0.5\phi V_{uc}$ no shear reinforcement is required except where the overall depth of the beam exceeds 750 mm, in which case minimum shear reinforcement shall be provided.

2. clause 8.2.5 (b): if $0.5\phi V_{uc} < V^* \leq \phi V_{u.\min}$ only minimum shear $A_{sv.\min}$ is required. Also according to the same standard, clause 8.2.5 (ii), if $V^* \leq \phi V_{uc}$ the minimum shear reinforcement requirement may be waived. According to clause 8.2.8:

   Equation 3-4

   $$A_{sv.\min} = 0.06\sqrt{f'_c}\, b_v s / f_{sy.f} \geq 0.35 b_v s / f_{sy.f}$$

   where:

   f'c is the concrete compressive strength, in this case 50 MPa
   b_v is the effective width of the web for shear
   s is the centre-to-centre spacing of shear fitments
   $f_{sy.f}$ is the characteristic yield strength of the reinforcement used as fitments

3. clause 8.2.5 (b): if $V^* > \phi V_{u.\min}$ shear reinforcement is required according to clause 8.2.10.

All these conditions are checked by the Matlab program `checkshear.m` (see Appendix C for code).

3.4 Finding the optimum topology using the SIMP method

The aim of topology optimisation is to determine the optimum layout of a structure subject to specific loads within a specific design domain. The topology optimisation method utilised in this study is the Solid Isotropic Material with Penalization, also known as SIMP or power-law method. The design variables are the relative densities of each finite element, and they relate to the material property via the power-law. The power law dictates that a relative change in one quantity results in a proportional relative change in the other quantity risen to a power.

The known quantities at the start of the optimisation will be the applied loads as determined in Section 3.2; the support conditions, which are assumed pinned (Transport and Main Roads 2010a, p. 3) (Standards Australia 2013, p. 24); the final volume of the structure; and the location and size of prescribed openings.

The loads will be applied on the top and sides of the culvert, as described in Section 3.2. However, to minimise computation time, only half of the culvert was modelled in the symmetric case and the culvert crown was assumed to be supported on rollers.

The final volume of the structure is one of the constraints of the topology optimisation method utilised in this study. The topology optimisation script will stop only when the total volume of the final structure is as required by the volume constraint, and when the variance of the relative densities is smaller than 0.5%, which is the chosen accuracy for the convergence criterion. Once the topology optimisation process finishes, the culvert with voids is modelled in Strand7 and the moment and shear capacity are determined. The reinforcement is placed as required in AS3600 and the void size and shape can be modified to allow for the placement of reinforcement within the specified cover. In this case, because the culverts are to comply with MRTS24, the minimum exposure classification is B2, which means the nominal cover is to be 35 mm (Standards Australia 2013, p. 16). This means that if the bar diameter is 20 mm, for example, the distance between the edge of the void and the edge of the culvert needs to be at least 35+35+20=90 mm.

Within the domain, an area representing the culvert nominal opening was assigned a relative density of 0.001, to ensure there is no material in the opening. This represents the second constraint.

The objective function is to determine the minimum vector of relative densities. The aim of the method is to find a topology with as many densities equal to 1 or 0, meaning no intermediate densities are desired since in this study only concrete and steel reinforcement will be utilised. The intermediate densities are penalised by being elevating to a power p. If p is too low or too high it can cause too many finite elements with intermediate densities or too fast a convergence to local minima (Sigmund & Maute 2013). Bendsøe and Sigmund (1999) claim that a p>3 will give results that have a physical meaning. The authors explain that if p can be modelled as a material if it complies with

Equation 3-5

$$p \geq \max\left\{\frac{2}{1-v}, \frac{4}{1+v}\right\}$$

where v is the Poisson ratio of the solid material.

However, this project will only utilise steel reinforcement and concrete and it will not attempt to model composite materials. Since the Poisson's ratio of the concrete utilised is 0.2 (as per AS3600-2009 Clause 3.1.5), Equation 3-5 would then become

Equation 3-6

$$p \geq \max\left\{\frac{2}{1-0.2}, \frac{4}{1+0.2}\right\} \therefore p \geq \max\{2.5, 3.33\}$$

Trying a penalisation factor of 4 resulted in non-convergence, so a penalisation factor of 3 was chosen instead, which gave good results.

The SIMP method was implemented by utilising a Matlab script, which is explained in detail in the next subsection.

3.4.1 Matlab script: `top_rcbc4.m`

To design the optimised culverts, a Matlab script called `top_rcbc4.m`, which was adapted from Sigmund (2001), was utilised. The adapted program can be found in Appendix F.

Firstly the material is uniformly distributed through the design domain, which is assumed to be rectangular with the finite elements assumed square like a quad4 element from Strand7. The scale of the real size to the modelled size culvert can vary, but in the majority of cases a scale of 25 proved sufficient. This means that each square finite element side represents 25 mm of the real culvert size. When the scale was diminished, the computation time was greatly increased to unpractical times without significantly improving the result, proving ineffective.

Because the box culverts need to have a certain size opening, a range of finite elements is made passive by changing their relative density to 1E-3. This means there is a void, not an element. If these relative densities were to be changed to zero it would result in a matrix singularity, hence the densities are changed to a very small number *ie* 1E-3.

Then the finite element analysis is performed to find the displacement vector U. To achieve that, the element stiffness matrix is generated, utilising the Young's modulus E and Poisson's ratio ν previously input into the code, which is this case translates into E=34800 MPa for our chosen 50 MPa concrete with ν=0.2. The global stiffness matrix can then be assembled by looping through all elements and by utilising element node numbers described as global element numbers to ensure correct placement of elements in the global stiffness matrix.

The next step is to apply forces to the RCBC model. The forces applied are those found by the load combination program, as described in more detail in Section 3.2 - Analysis of SM1600 loads. The force applied in the program as written by Sigmund (2001) was a unit concentrated force at the edge of the design domain. This was changed in the adapted code to model the loads the RCBCs were subjected to. The vertical load on top of the crown is always a uniformly distributed load (UDL), so the point force was changed to a vector to model this. The side load was approximated to a UDL to facilitate implementation in Matlab.

Subsequently, the support conditions must be set up. Every element has two degrees of freedom, namely horizontal and vertical. To implement a support, these degrees of freedom are eliminated from the linear equations to model the constrained degrees of freedom. The unconstrained degrees of freedom are the difference between all degrees of freedom and the fixed ones.

After that the objective function, which is the minimum vector of relative densities, is found by applying finite element analysis principles, yielding

Equation 3-7 - Optimisation objective function

$$\min_{x} : c(\mathrm{x}) = \mathrm{U}^T \, KU = \sum_{e=1}^{N} (x_e)^p u_e{}^T k_0 u_e$$

where

x is the vector of relative densities, the design variables
x_{min} is the minimum of the vector of relative densities, with non-zero values
U is the global displacement vector
K is the global stiffness matrix
u_e is the element displacement vector and
k_e is the element stiffness matrix

It can be seen that the objective function is found by multiplying the global force matrix, which is F=KU, by the transposed global displacement matrix U^T. The summation displayed in the right side of Equation 3-7 is then implemented in the Matlab script.

Sigmund (2001) claims that it is possible to improve the likelihood of the existence of solutions by implementing a filtering scheme, which is the next step in the Matlab script. The filter modifies the element sensitivities by using a convolution operator (weight factor) $\hat{H}_f = r_{\min} - dist(e, f)$ in which r_{min} is the filter size divided by the element size and *dist(e,f)* is the distance between the centre of element e to the centre of element f. Sigmund (2001) warns that the filter does not guarantee the existence of solutions, but it has been tested by the author in various applications with positive results.

The design variables stored in the x vector are then updated using the optimality criteria method. To do this, the value of the Lagrange multiplier that satisfies the volume constraint chosen by the user when the function is called is found. The bi-sectioning method is utilised to achieve this, since the material volume is a monotonously decreasing function of the Lagrange multiplier (Sigmund 2001).

Each vector of design variables is then printed as an image in turn using a black-white colour map, in which black means relative densities of 1 (presence of material) and white means relative densities of 1E-3 (voids). Areas with grey colour would indicate a composite material of intermediate density.

Each iteration summary is also printed on the screen with the iteration number, the objective function value, the fraction of the initial volume, the convergence criterion and the time the iteration was performed.

With this information it is possible to identify where the voids should be and how the optimisation was performed.

The program is run in Matlab by calling it from the prompt line with:

```
[x,U]=top_rcbc4(span,legheight,volfrac,penal,rmin)
```

In square brackets are the program's outputs, which will be the vector of relative densities, x, and the global displacement matrix U. The inputs are in parenthesis:

`span` is the RCBC span in mm
`legheight` is the RCBC leg height in mm
`volfrac` is the volume constraint *ie* 30% is entered as 0.30
`penal` is the penalisation factor
`rmin` is the filter size divided by the element size

3.5 Feasibility analysis

Once the optimum culvert has been found, an analysis was be carried out to ascertain its constructability and commerciality by investigating the level of efficiency and cost savings during design, manufacture and installation.

The cost information was gathered by interviewing Mr. Roome, an Engineered Solutions Manager with over 20 years private industry experience and vast knowledge of RCBCs.

3.5.1 Production Costs of an RCBC

When performing a cost estimation of a given RCBC, the factors taken into consideration are:

- Concrete materials: cement, aggregate, water
- Reinforcement materials: steel
- Labour: preparing steel cage, casting procedures, loading procedures, quality assurance (QA) checks
- Overhead costs: plant, asset depreciation, maintenance, staff rates
- Design costs

The customer selling price will then be this total production cost plus a profit margin. According to Roome (2014), the private industry normally offers delivery as a service to the customer and small margins are added to delivery to cover the administration costs regarding its organisation. Alternatively, the precast product can be picked up from the factory, which is called 'ex-works' and does not involve extra costs. Delivery will therefore not be included in this feasibility analysis.

3.5.2 Estimating Procedure

There are various ways to measure costs. Materials, for example, are normally measured in \$/tonne or \$/m^3. Labour and overhead costs are normally measured in man-hours/tonne. For instance, in the case of labour, if the cost is 4 man-hours/tonne and the product weighs 1 tonne, there were 4 hours of labour activity involved to produce it. This activity includes setting up the moulds, producing the steel cages, casting the product, the curing procedure, loading and checking. Design is normally charged in \$/hour.

To estimate the cost of a product, all these costs need to be taken into consideration and transformed into the same currency *ie* dollars. The following sections will look at each cost component in detail.

3.5.2.1 Design

The cost of design will vary from company to company. In this study, it is assumed a design engineer with a couple of years' experience will design the culverts, utilising software as it is common in the industry. To estimate the design cost to be input into the RCBC cost estimate, let us assume this engineer earns \$80000 per year and works 38 hours per week. That would give the company a cost of approximately \$40.50 per hour to pay for this engineer's salary. However, for the engineer to design the box culverts, it needs an office, computers, software and the cost to maintain all this and the depreciation of all this needs to be taken into account. In this study, it is assumed the cost of an engineer's hour to design a box culvert, in total, is approximately $80/hour, utilising design software.

A design is produced according to the required specifications and it yields the product mass and reinforcement content, which are the input in the cost analysis. More routine designs are done more quickly and special designs or non-routine requirements will increase design time.

3.5.2.2 Materials

The cost of materials, according to Roome (2014), does not vary too much since the production procedures are standard throughout the industry and a supplier cannot generally get the same quality product for a very different price. Mr. Roome believes the cost of concrete is around $140/m^3 and the cost of reinforcement steel is $1300/tonne. These values will be utilised to estimate the cost of materials.

3.5.2.3 Labour costs

On average, one hour of labour costs around $40. The most significant component of a large box culvert price is labour, since it is the one that can vary the most and that is significantly large compared to other costs. This means that, in the precast industry, one of the most effective ways to save on production costs is to save on labour costs. This is achieved by simplifying procedures like casting and reinforcement cage manufacture and augmenting their level of repetitiveness.

One way to achieve this is by maximising the amount of units made in the same size. That is because making a lot of units utilising the same mould setup will spread the cost of setting up that mould onto more units, with the setup cost for each unit decreasing. For example, a large RCBC job normally consists of an average of 600 metres worth of box culverts. Large box culverts are normally sold in 2.4 m lengths, since it is more efficient to produce them than the 1.2 m lengths. This is due to the fact that the reinforcement cage, mould setup and casting procedures have to be done once only to product 2.4 m of product, while they would have to be done twice to produce the same length using a 1.2 m long mould.

Roome (2014) advises that in his experience, the private industry estimates costs by calculating the quantity of man-hours required to product a tonne of product. For example, the price of large box culverts in Mr Roome's experience is around 2.5 man-hours per tonne, if the design is standard, without additions, voids or special requirements and if the culvert is transported legs down.

The loading and transportation procedure is different for culverts with legs up to 2.1 metres and those with taller legs. If the product's leg is up to 2.1 m long, it can generally be transported legs down on the truck. This means there is no rotation involved in demoulding, loading, unloading and installing.

If the culvert leg is taller than 2.1 m a design analysis will have to be carried out that takes into consideration the fact that the centre of gravity of the product will be higher in the truck and that creates a much higher risk for transportation. The transportation design analysis generally yields one of two possible solutions: either the culvert is transported upside down, with the crown on the truck bed, or it is transported on its side. Either of these will incur extra costing related to design, labour for the extra rotations and extra lifters setup required, as well as longer loading times.

3.5.2.4 Overheads

The term overheads refers to the costs of operating a business. They include plant depreciation and maintenance, rent, water, electricity, insurance, employees' salaries, payroll taxes, employee pension costs and other employee benefits.

Roome (2014) estimates these costs to be around $70/tonne at present at his place of employment. This will obviously vary depending on how a business is run, how modern their plant are, how much maintenance everything needs, where the factory is located, among many others. However, for this study, the value used for overheads is \$70/tonne.

3.6 Conclusion

The methodology for this project consists of four main steps: analysis of loads, design of the standard RCBC, determination of optimum culvert and feasibility analysis Firstly, the SM1600 loads were analysed and the worst case for each part of the culvert were found and taken as design load. This was achieved by implementing a couple of Matlab scripts, namely `load_comb.m` and `finalscript.m`. Then, a flexibility and shear analysis was carried out to design the reinforcement according to AS1597.2-2013 and MRTS24 (Aug/11). This non-optimised culvert was the basis for a comparison with the optimum RCBC. The Matlab scripts utilised to implement that were `flexanalysis.m`, `devlength.m` and `checkshear.m`. Following that the optimum topology for the RCBC under the design loads was found utilising the SIMP method and finite element analysis. The Matlab script utilised in this step was `top_rcbc4.m`, which was an adaptation from Sigmund (2001). Lastly, a feasibility analysis was carried out to outline the benefits and drawbacks of utilising this optimisation procedure in the industry. The information was obtained by means of an interview with an experienced manager in the industry.

CHAPTER 4 - RCBC DESIGN

4.1 Introduction

The structural design procedure is always iterative. Firstly the leg and crown thicknesses are assumed. Then the design loads are found and applied to the finite element model of the structure generated in Strand7. The bending moment and shear force diagrams are generated. A flexibility and shear analysis then follows to find a suitable reinforcement for the structure. If by any chance the section is found to be too thin and fails in shear or bending, the assumed values at the beginning of the procedure are changed and the process starts again. There are many ways a compliant design can be achieved and different designers could find different acceptable solutions. The designs found in this study were kept as similar as feasibly possible to each other to allow for easy comparison.

4.2 Load Combination Results

Four horizontal and four vertical loads are applicable to this study, as discussed in Section 3.2. However, these loads may be combined in a variety of ways to model different scenarios. That is why different load factors apply to each load, and the aim is to find out which load combination is the worst so that it can be used as the design load.

To achieve that, a Matlab script named `load_comb.m` was developed. It puts together all possible combinations of loads with their applicable load factors, in both the symmetric and asymmetric loading cases, which are discussed in detail in the following sections. The Matlab script can be found in Appendix G.

4.2.1 Symmetric loading

Following the principles outlined in Section 3.2, the only possible symmetric load combinations for vertical and horizontal loads respectively are shown in Figure 4-1 and Figure 4-2.

1. $1.0 \cdot W_{DC} + 1.4 \cdot W_{FV} + 0 \cdot W_{CV} + 1.8 \cdot W_{LV}$
2. $1.0 \cdot W_{DC} + 1.4 \cdot W_{FV} + 1.5 \cdot W_{CV} + 0 \cdot W_{LV}$

3. $1.0 \cdot W_{DC} + 0.9 \cdot W_{FV} + 0 \cdot W_{CV} + 1.8 \cdot W_{LV}$
4. $1.0 \cdot W_{DC} + 0.9 \cdot W_{FV} + 1.5 \cdot W_{CV} + 0 \cdot W_{LV}$

5. $1.0 \cdot W_{DC} + 0.9 \cdot W_{FV} + 0 \cdot W_{CV} + 0 \cdot W_{LV}$
6. $1.0 \cdot W_{DC} + 1.4 \cdot W_{FV} + 0 \cdot W_{CV} + 0 \cdot W_{LV}$

Figure 4-1 - Vertical load combinations

7. $0.7 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 1.8 \cdot W_{LH}$
8. $0.7 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 1.8 \cdot W_{LH}$
9. $1.4 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 1.8 \cdot W_{LH}$
10. $1.4 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 1.8 \cdot W_{LH}$

11. $0.7 \cdot W_{FH} + 0.7 \cdot W_{AH} + 1.5 \cdot W_{CH} + 0 \cdot W_{LH}$
12. $0.7 \cdot W_{FH} + 1.4 \cdot W_{AH} + 1.5 \cdot W_{CH} + 0 \cdot W_{LH}$
13. $1.4 \cdot W_{FH} + 0.7 \cdot W_{AH} + 1.5 \cdot W_{CH} + 0 \cdot W_{LH}$
14. $1.4 \cdot W_{FH} + 1.4 \cdot W_{AH} + 1.5 \cdot W_{CH} + 0 \cdot W_{LH}$

15. $0.7 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$
16. $0.7 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$
17. $1.4 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$
18. $1.4 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$

Figure 4-2 - Horizontal load combinations

It can be seen that live construction loads and live roadway loads are never considered together. That is because unless a specific construction plant vehicle is utilised, the culverts are to be designed to support construction traffic loads and their effects defined by the wheel loadings of SM1600 (Standards Australia 2013, p. 28). This means the construction plant cannot be heavier than the SM1600 traffic loads the culverts are being designed for. However, the load factor for the construction loads and their effects is 1.5 while the one for live roadway traffic is 1.8. This is to account for the fact that the construction load will happen less often than the roadway live load.

To find all possible symmetric load combinations, we assemble each of the vertical loads (from 1 to 6) with each of the horizontal loads (from 7 to 18), keeping in mind that if there is no vertical construction load there cannot be a horizontal construction load. The same applies to roadway live load.

Combination 1:	1V and 7H
Combination 2:	1V and 8H
Combination 3:	1V and 9H
Combination 4:	1V and 10H
Combination 5:	2V and 11H
Combination 6:	2V and 12H
Combination 7:	2V and 13H
Combination 8:	2V and 14H
Combination 9:	3V and 7H
Combination 10:	3V and 8H
Combination 11:	3V and 9H
Combination 12:	3V and 10H
Combination 13:	4V and 11H
Combination 14:	4V and 12H
Combination 15:	4V and 13H
Combination 16:	4V and 14H
Combination 17:	5V and 15H
Combination 18:	5V and 16H
Combination 19:	5V and 17H
Combination 20:	5V and 18H
Combination 21:	6V and 15H
Combination 22:	6V and 16H
Combination 23:	6V and 17H
Combination 24:	6V and 18H

4.2.2 Asymmetric loading

The only direction in which the symmetric and asymmetric loadings differ is the horizontal, and the differences are only relevant regarding fill and compaction, since the construction and roadway load depend only on the vertical loads, which remain the same. Therefore, on one side of the culvert there will be

17. $1.4 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$
18. $1.4 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$

Figure 4-3 – Asymmetric horizontal load combinations – one side of culvert

The correspondent loads on the other side of the culvert will be:

15. $0.7 \cdot W_{FH} + 0.7 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$
16. $0.7 \cdot W_{FH} + 1.4 \cdot W_{AH} + 0 \cdot W_{CH} + 0 \cdot W_{LH}$

Figure 4-4 – Asymmetric horizontal load combinations – other side of culvert

The only possible asymmetric load combinations will therefore be:

Combination 1:	1V and 16H/17H
Combination 2:	1V and 16H/17H
Combination 3:	1V and 15H/18H
Combination 4:	1V and 16H/18H
Combination 5:	2V and 15H/17H
Combination 6:	2V and 16H/17H
Combination 7:	2V and 15H/18H
Combination 8:	2V and 16H/18H
Combination 9:	3V and 15H/17H
Combination 10:	3V and 16H/17H
Combination 11:	3V and 15H/18H
Combination 12:	3V and 16H/18H
Combination 13:	4V and 15H/17H
Combination 14:	4V and 16H/17H
Combination 15:	4V and 15H/18H
Combination 16:	4V and 16H/18H

Combination 17: 5V and 15H/17H
Combination 18: 5V and 16H/17H
Combination 19: 5V and 15H/18H
Combination 20: 5V and 16H/18H

Combination 21: 6V and 15H/17H
Combination 22: 6V and 16H/17H
Combination 23: 6V and 15H/18H
Combination 24: 6V and 16H/18H

However, it is important to note that the objective of this study is to analyse culverts that comply with MRTS24 and can be installed under main roads. If the horizontal loading on the culvert due to fill and compaction is asymmetric, it means there is a different amount of fill on either side of the culvert or that one side is unsuitably compacted while the other is suitably compacted. Because this situation is very rare in Main Roads projects and it would probably not be compliant, the asymmetric loading is not going to be considered in this study.

A programme called `finalscript.m` (see Appendix D) was created in MATLAB to reveal the critical symmetrical load combination, after the script `load_comb.m` (see Appendix G) generates all possible combinations. The critical load combination was found by analysing the various possible combinations of vehicles, load distributions through fill and fill heights in increments of 0.1 m.

4.3 Design Loads

As explained in Chapter 3 - Methodology, the culverts in this study are assumed to be subjected to SM1600 loads, which represent the W80, A160, M1600 and S1600 design loads. (Standards Australia 2013, p. 58). These loads model road traffic design loads for main and secondary roads, which are commonly specified in the industry. Heavy load platform loads (HLP320 and HLP400) and railway loads (300LA) are not included in this study.

4.4 1815 RCBC Design

To design the RCBC with 1.8 m span and 1.5 m leg (1815 RCBC), different crown thicknesses (200 mm, 300 mm and 400) and leg thicknesses (200mm, 220mm, 300 mm and 350mm) were trialled before the design could be finalised. The detailed calculations for a trial design with a leg thickness of 200 mm and crown thickness of 250 mm can be seen in Appendix H. The design called for shear reinforcement with N12 bars at 33 mm centres, meaning the spacing between the edge of the bars would actually be 19 mm. That is a problem since most 50MPa concrete mixes would have maximum aggregate size of 20 mm, and having those bars close together would impact with the casting procedure.

The most suitable design for the 1815 RCBC was achieved with a 350 mm leg and 400 mm crown, meaning the overall culvert width was 2.5 m and the overall height was 1.9 m. The results from the load combination Matlab script `finalscript.m` (see appendix D) were as shown in Figure 4-5 and Figure 4-10. The loads on the top of the culvert are uniformly distributed over the entire top of culvert, as expected since the truncated prism model (see Section 3.2 for details) distributes the vehicle and construction loads uniformly over the top of the culvert. The other loads that act on top of the culvert are also uniformly distributed, namely the fill and the self-weight. There are 20 straight lines in Figure 4-5, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 654 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 114.4 kPa.

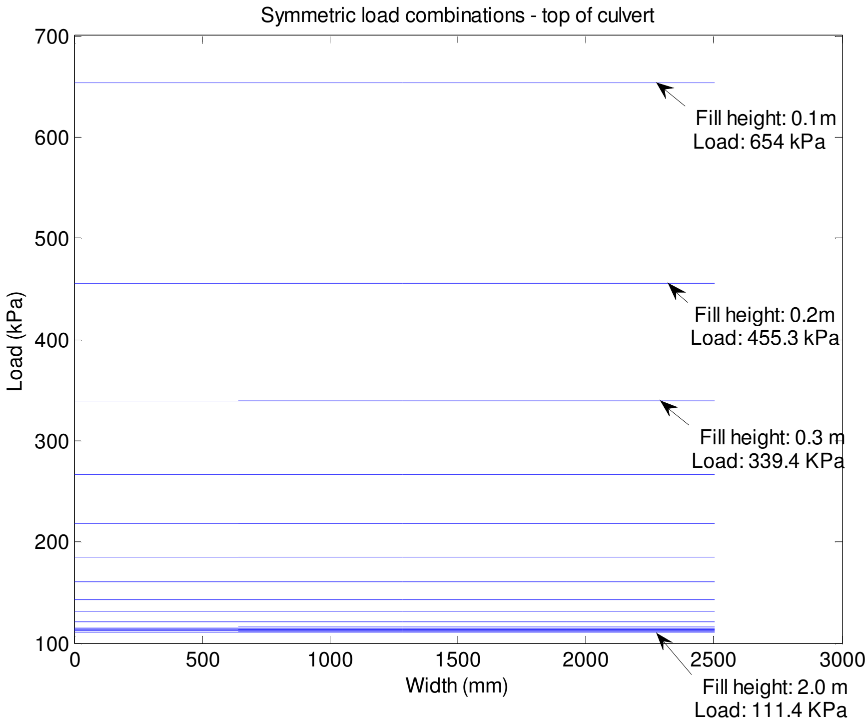

Figure 4-5 – Loads on top of 1815 RCBC

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors. All vertical load values for the various fill heights as plotted in Figure 4-5 can be seen in Table 4-1, since the Matlab script `finalscript.m` calculates all possible load combinations in 0.1 m fill increments:

Table 4-1 - Vertical loads on 1815 RCBC

Fill Height (m)	Load (kPa)
0.1	654.039
0.2	455.3062
0.3	339.4182
0.4	266.775
0.5	217.5766
0.6	184.623
0.7	160.7142
0.8	142.2502
0.9	131.031
1	121.6566
1.1	115.927
1.2	110.2422
1.3	111.8022
1.4	111.607
1.5	111.4566
1.6	111.351
1.7	113.0902
1.8	113.0742
1.9	114.7966
2	114.4166

The horizontal loads have a different form to the vertical loads (Figure 4-6). They increase from the top of culvert up to 0.5 m below the top of culvert, from which they continue uniformly. That is because the only horizontal load which is uniformly distributed is the horizontal live load W_{LH} as shown in Figure 4-7. The compaction load W_{AH} is shown in Figure 4-8. As dictated in AS1597.2-2013, it increases up to 0.5 m below the top of the culvert, then remains constant up to 1.5 m below the top of the culvert, then decreases linearly up to 2.0 m below the top of the culvert (see Section 3.2.9 for details). The horizontal fill load W_{FH} is also non-uniform since it varies with the depth below the culvert.

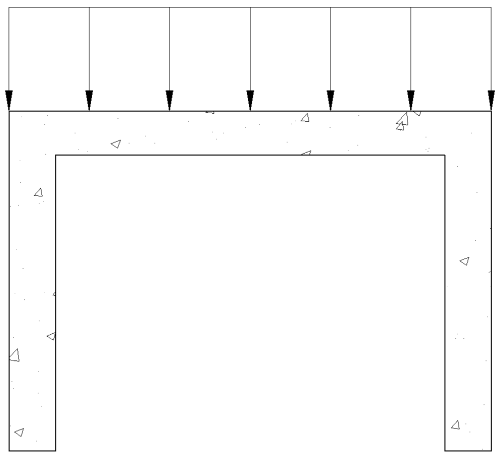

Figure 4-6 - Vertical loads

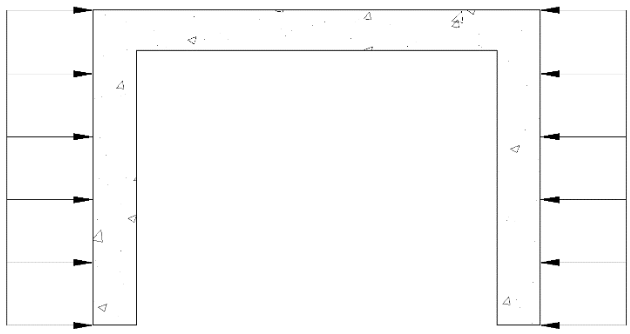

Figure 4-7 - Horizontal live load

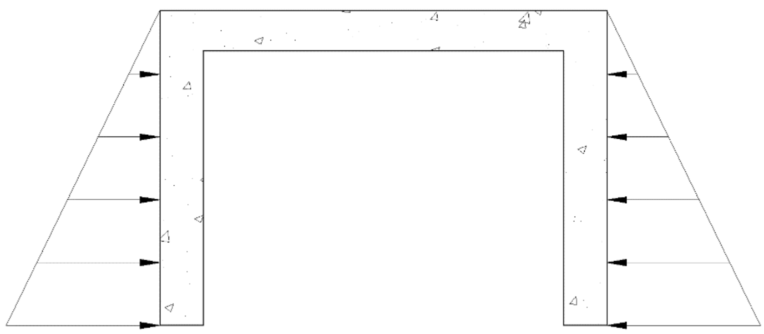

Figure 4-8 - Compaction load

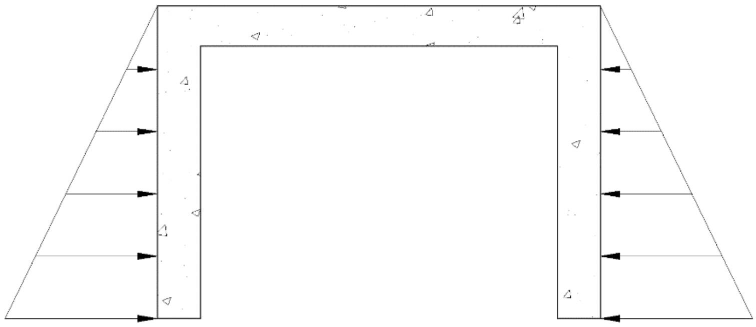

Figure 4-9 - Horizontal fill load

Because of these non-uniformities, the shape of the horizontal load graphs (Figure 4-10) differs from the vertical load graphs (Figure 4-5).

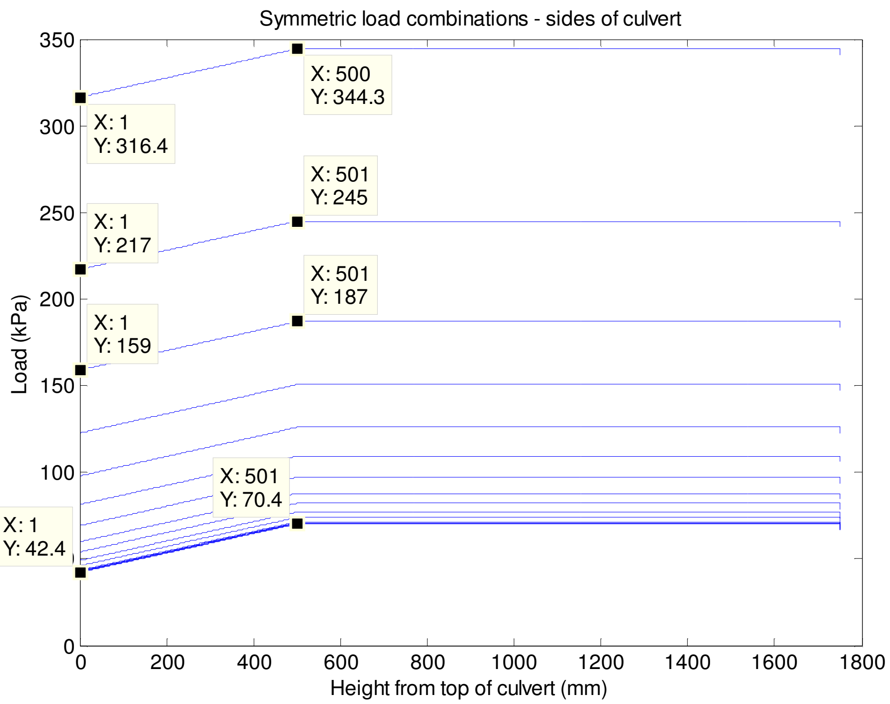

Figure 4-10 - Loads on the side of 1815 RCBC

The 1815 RCBC with crown thickness of 400 mm and the leg thickness of 350 mm is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-11 and Figure 4-12.

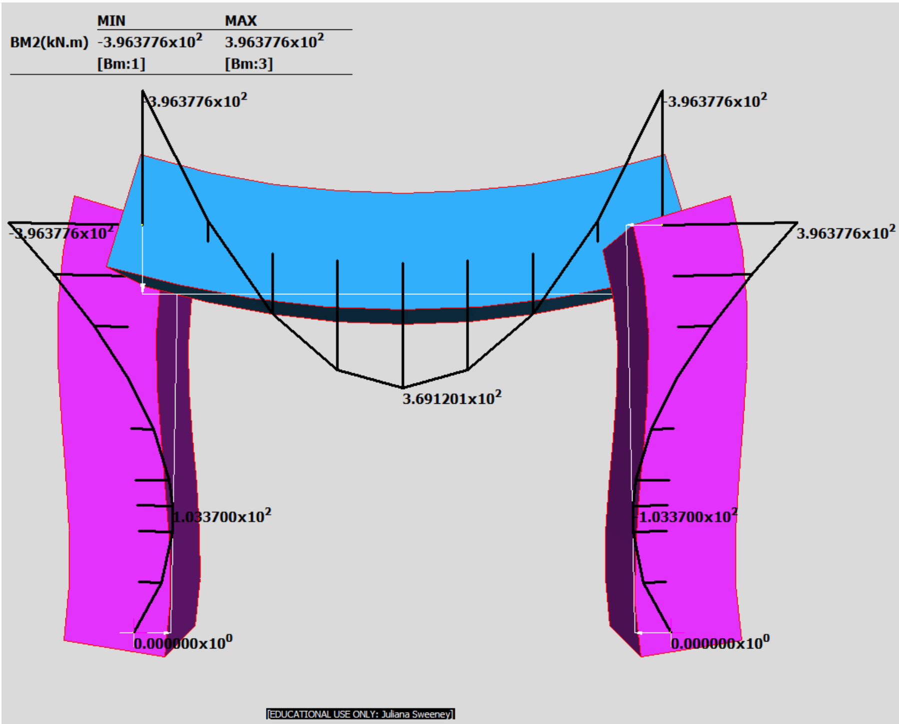

Figure 4-11 – Bending moment diagram for 1815 RCBC

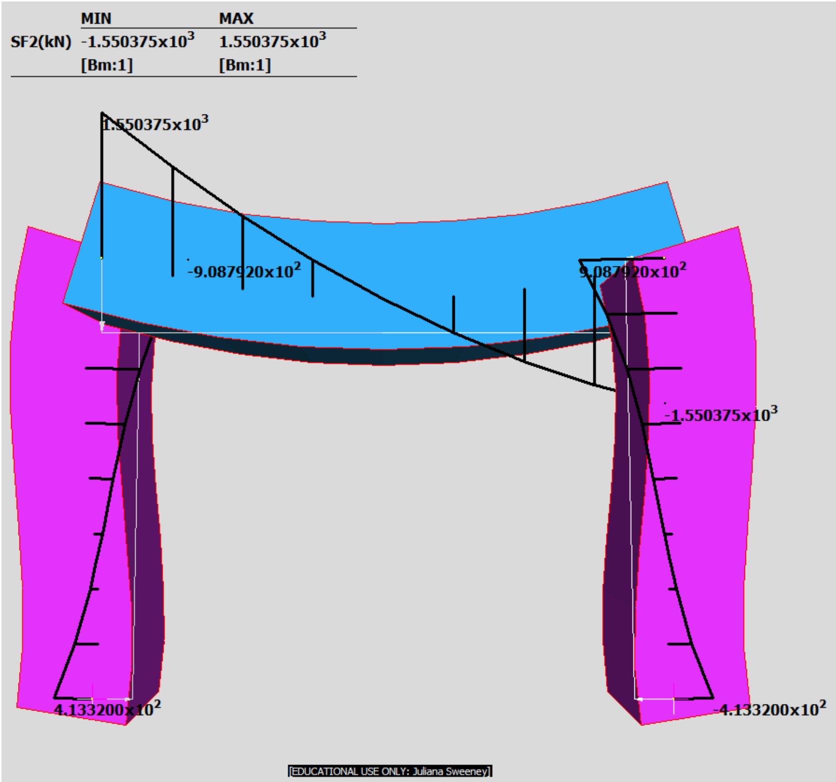

Figure 4-12 - Shear force diagram for 1815 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in Section 3.3.5 - Non-optimised Culverts and Section 3.3.6 - Reinforcement.

Flexure and shear analysis for the middle of the crown

For the middle of the crown, M*=369.1 kNm (Figure 4-11) and by running `flexanalysis.m` with 14-N16 bars as tensile reinforcement, the results are:

```
>> flexanalysis(400,14,16,0,12) ;
dn =    19.6200
ku =     0.0550
Cc =    1.4009e+03
Cs =      0
Mu =   490.4901
```

```
phiMu =  392.3921
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 14
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 14-N16 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 464$ mm. This means this bar has to be at least 928 mm long.

The shear for the middle of the crown is 0 so a shear check does not need to be carried out and no shear reinforcement is required.

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg.

<u>Flexure and shear analysis for end of the crown</u>

The design moment at the end of the crown is M*=396.4 kNm (see Figure 4-11). By running `flexanalysis.m` for 15-N16 bars in tension, the results are:

```
>> flexanalysis(400,15,16,0,12) ;
dn =   21.0200
ku =    0.0589
Cc =   1.5008e+03
Cs =     0
Mu =  524.7540
phiMu =  419.8032
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 15
```

This means that:

- the section is under-reinforced since $k_u<0.36$;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

Therefore, a shear check is required to check if shear reinforcement is necessary. The design shear force at the end of the crown is V*=1550.4 kN (see Figure 4-12). By running `checkshear.m` for the 15-N16 bars in tension, the results are:

```
>>checkshear(1550.4E3,16,15,400);
Vumax =      8568000
Vuc =   6.5536e+05
Vumin =   1.2612e+06
Vusmin =   1.5595e+06
s_vusmin =   88.1342
s =   66.6667
Shear reinforcement is required. Provide 7-N12 bars at
67 mm spacings
```

This means that $(1550400 = V^*) > (\phi V_{u.\min} = 882840)$ and 7-N12 ligatures will be provided at 67 mm spacing for an extent of D=400 mm. The maximum spacing, the one relating to $V_{us.min}$ noted in the Matlab script as `s_vusmin`, would be 88 mm therefore 67 mm spacing is suitable. If 6-N12s were chosen instead, that spacing would be more than the minimum and it would not be suitable.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 15-N16 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 464$ mm. This means that each leg of this L bar this bar has to be at least 464 mm long.

Flexure and shear analysis for top of the leg

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=396.4 kNm (see Figure 4-11). However, this section is thinner since the leg is 350 mm and the crown is 400 mm. By running `flexanalysis.m` for 18-N16 bars in tension, the results are:

```
>>flexanalysis(350,18,16,0,12) ;
dn =   25.2300
ku =    0.0822
Cc =   1.8014e+03
Cs =     0
Mu =  537.1291
phiMu =  429.7033
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 18
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=908.8 kN (see Figure 4-12). By running `checkshear.m` for the 18-N16 bars in tension, the results are:

```
>>checkshear(908.8E3,16,18,350)
Vumax =     7368000
Vuc =   6.5511e+05
Vumin =   1.1761e+06
Vusmin =   6.4317e+05
s_vusmin =  131.2630
s =   87.5000
Shear reinforcement is required. Provide 5-N12 bars at
88 mm spacings
```

This means that $(908800 = V^*) > (\phi V_{u.\min} = 823270)$ and 5-N12 ligatures will be provided at 88 mm spacing for an extent of D=350 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 18-N16 in tension, with cover=35 mm and k1=1.3 is $L_{sy.tb} = 603$ mm. This means that each leg of this L bar this bar has to be at least 603 mm long.

Flexure and shear analysis for bottom of the leg

The design moment at the bottom third of the leg is M*=103.37 kNm (see Figure 4-11). By running `flexanalysis.m` for 8-N12 bars in tension, the results are:

```
>> flexanalysis(350,8,12,0,12) ;
dn =   6.1600
ku =    0.0199
Cc =  439.8240
Cs =      0
Mu =  134.9574
phiMu =  107.9659
Number of N12 bars required for compression:  0
Number of N12 bars required for tension:  8
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the bottom of the leg is V*=413.32 kN (see Figure 4-12). By running `checkshear.m` for the 8-N12 bars in tension, the results are:

```
>>checkshear(413.32E3,12,8,350)
Vumax =      7416000
Vuc =   4.1076e+05
Vumin =   9.3515e+05
ans =Minimum shear reinforcement is required.
```

This means that $0.5\phi V_{uc} < V^* \le \phi V_{u.\min}$ and $A_{sv.\min}$ will be provided for an extent of D=350 mm. In this case, $V^* > \phi V_{uc}$ so the minimum shear reinforcement requirements cannot be waived. For it to be waived, 24-N12s would have to be provided and since there is a great difference between the required 8-N12s and 24-N12s, shear ligatures will be provided and 8-N12 bars will be installed.

Using the maximum spacing of s=0.5D=175 mm from AS3600-2009 clause 8.2.12.2 into Equation 3-4, the result is $A_{sv.\min} = 356.3818$ mm, which means there will be 4-N12 ligatures required at 116 mm spacings.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 8-N12 bars in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 348$ mm. This means this bar has to be at least 696 mm long.

The final 1815 RCBC reinforcement is shown diagrammatically in Figure 4-13.

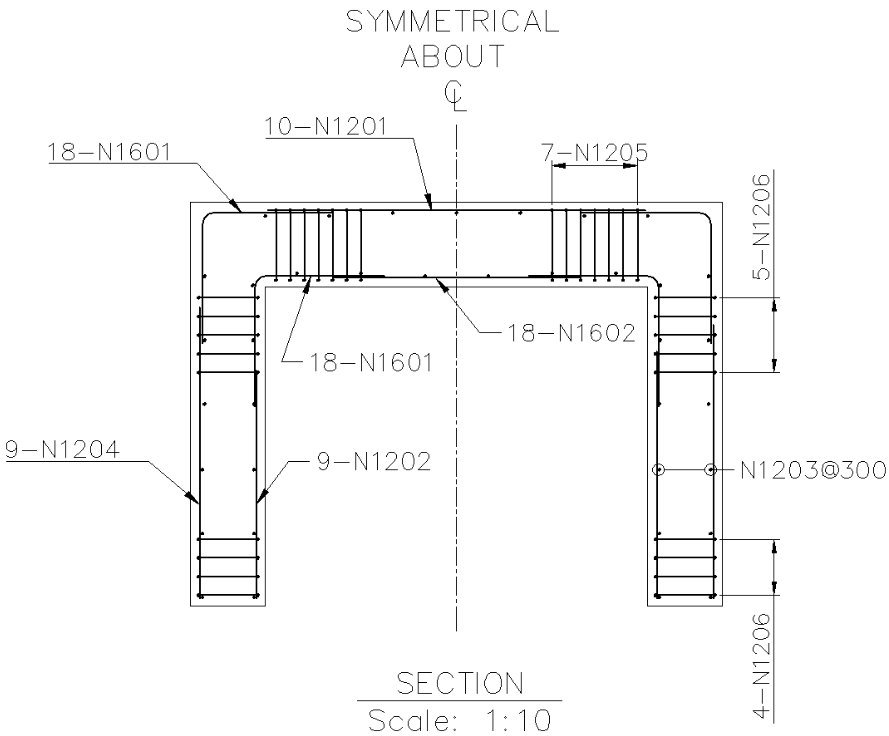

Figure 4-13 - 1815 RCBC reinforcement

In order to simplify the assembly of the reinforcement cages, especially with relation to the ligatures, the L bars at the end of the crown / top of the leg (N1601) were made the same length with the same spacing. The bar at the bottom of the crown (N1602) was also changed from 14-N16 to 18-N16, to follow the same spacing as the L bars and therefore facilitate the installation of the ligatures. The quantity of the bar N1204 was changed from 8 to 9 to facilitate the placement of ligatures, which should not be further apart than 600 mm through the width of the beam. The bars N1201 and N1204 were included to enable the connection of the distribution bars for cracking and the shear ligatures.

The reinforcement schedule for the 1815 RCBC is shown in Table 4-2.

Table 4-2 - 1815 RCBC reinforcement schedule

Bar Mark	Grade & Size	Qty	Total mass (kg)	Shape
N1201	N12	10	15.77	straight
N1202	N12	18	17.19	straight
N1203	N12	37	76.56	straight
N1204	N12	18	21.90	straight
N1205	N12	70	113.82	Ligs
N1206	N12	72	94.18	Ligs
N1601	N16	72	136.18	L
N1602	N16	18	33.00	straight
	Total reinforcement mass:		508.60	kg

4.5 1818 RCBC Design

To design the RCBC with 1.8 m span and 1.8 m leg (1818 RCBC), the same cross section was utilised than that of the 1815 RCBC (see Section 4.4), that is, a 350 mm leg and 400 mm crown. The overall culvert width was therefore 2.5 m and the overall height was 2.2 m.

The results from the load combination Matlab script `finalscript.m` (see appendix D for code) were as shown in Figure 4-14 and Figure 4-15. The loads on the top of the culvert are uniformly distributed over the entire top of culvert, as expected since the truncated prism model (see Section 3.2 for details) distributes the vehicle and construction loads uniformly over the top of the culvert. The other loads that act on top of the culvert are also uniformly distributed, namely the fill and the self-weight. There are 20 straight lines in Figure 4-14, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 656.2 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 112.4 kPa.

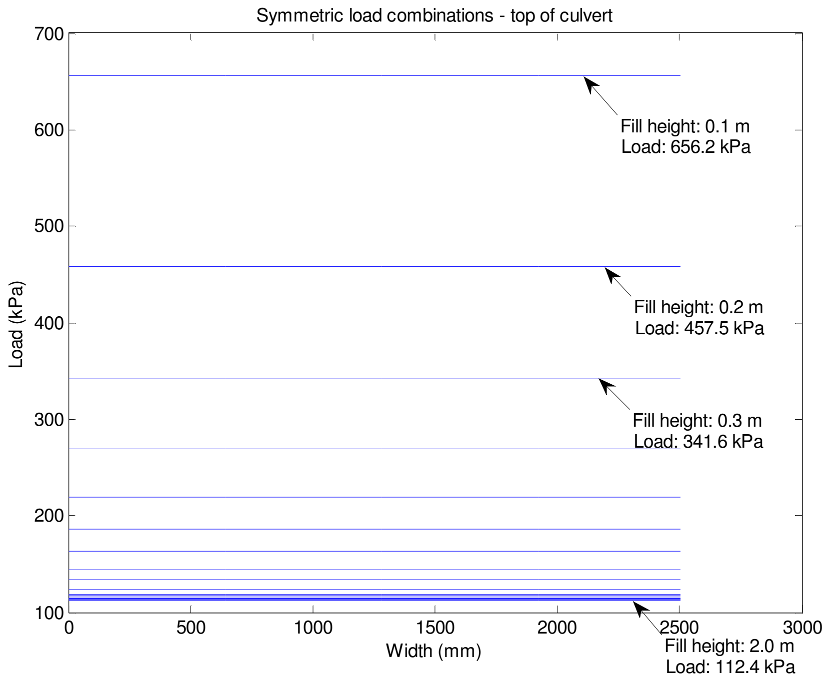

Figure 4-14 – Loads on top of 1818 RCBC

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors.

The horizontal loads have a different form to the vertical loads (see Section 4.4). Because of non-uniformities, the shape of the horizontal load graphs differs from the vertical load graphs.

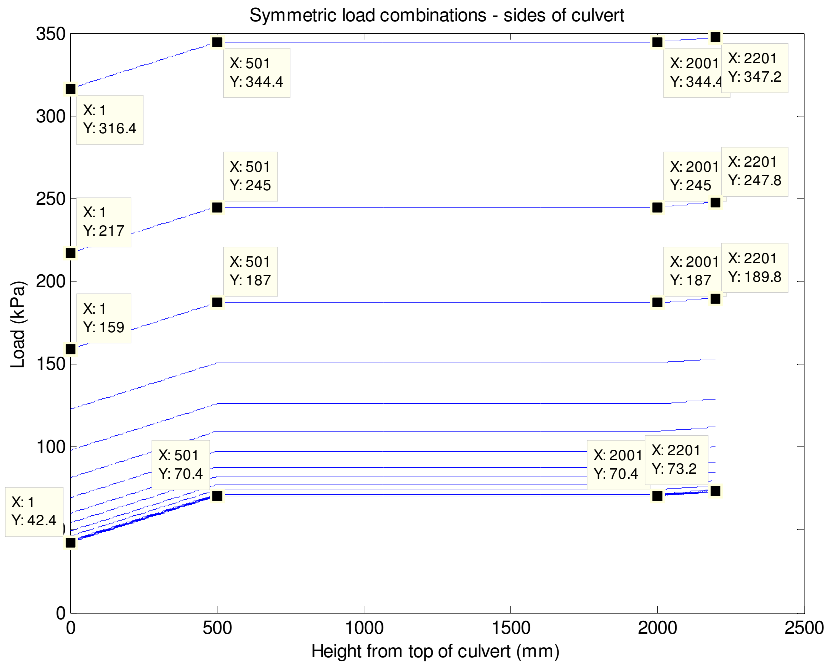

Figure 4-15 - Loads on the side of 1818 RCBC

The 1818 RCBC with crown thickness of 400 mm and the leg thickness of 350 mm is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-16 and Figure 4-17.

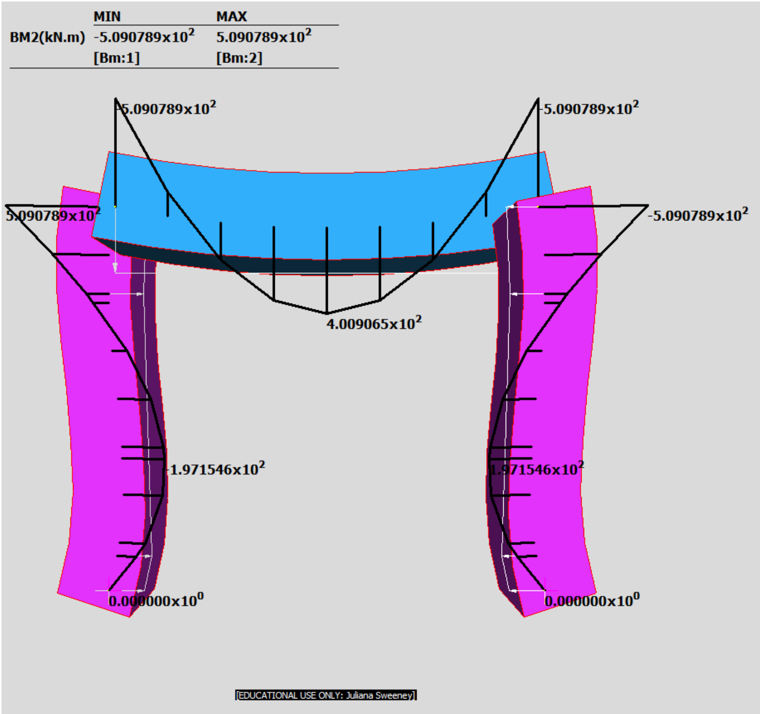

Figure 4-16 – Bending moment diagram for 1818 RCBC

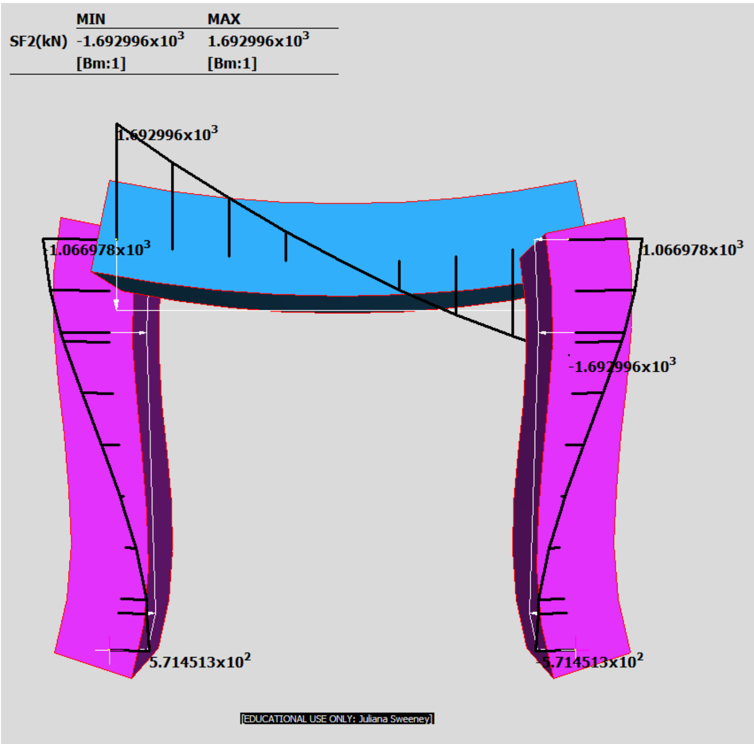

Figure 4-17 - Shear force diagram for 1818 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in Section 3.3.5 - Non-optimised Culverts.

Flexure and shear analysis for the middle of the crown

For the middle of the crown, M*=400.91 kNm (Figure 4-16) and by running `flexanalysis.m` with 15-N16 bars as tensile reinforcement, the results are:

```
>> flexanalysis(400,15,16,0,12) ;
dn =   21.0200
ku =    0.0589
Cc =   1.5008e+03
Cs =      0
Mu =  524.7540
phiMu =  419.8032
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 15
```

This means that:

- the section is under-reinforced since $k_u<0.36$;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 15-N16 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 464$ mm. This means this bar has to be at least 928 mm long.

The shear for the middle of the crown is 0 so a shear check does not need to be carried out and no shear reinforcement is required.

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg.

Flexure and shear analysis for end of the crown

The design moment at the end of the crown is M*=509.1 kNm (see Figure 4-16). By running `flexanalysis.m` for 19-N16 bars in tension, the results are:

```
>> flexanalysis(400,19,16,0,12) ;
dn =   26.6300
ku =    0.0746
Cc =   1.9014e+03
Cs =     0
Mu =  661.0715
phiMu =  528.8572
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 19
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The design shear force at the end of the crown is V*=1693 kN (see Figure 4-17). By running `checkshear.m` for the 19-N16 bars in tension, the results are:

```
>> checkshear(1693E3,16,19,400);
Vumax =     8568000
Vuc =   7.0909e+05
Vumin =   1.3149e+06
Vusmin =   1.7095e+06
s_vusmin =   80.4016
s =   66.6667
```

```
Shear reinforcement is required. Provide 7-N12 bars at
67 mm spacings
```

This means that $(1693000 = V^*) > (\phi V_{u.\min} = 920430)$ and 7-N12 ligatures will be provided at 67 mm spacing for an extent of D=400 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 19-N16 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 464$ mm. This means that each leg of this L bar this bar has to be at least 464 mm long.

<u>Flexure and shear analysis for top of the leg</u>

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=509.1 kNm (see Figure 4-16). However, this section is thinner since the leg is 350 mm and the crown is 400 mm. By running `flexanalysis.m` for 23-N16 bars in tension, the results are:

```
>>flexanalysis(350,23,16,0,12) ;
dn =   32.2400
ku =    0.1050
Cc =   2.3019e+03
Cs =     0
Mu =  680.7193
phiMu =  544.5754
Number of N12 bars required for compression:  0
Number of N16 bars required for tension: 23
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=1693 kN (see Figure 4-17). By running checkshear.m for the 23-N16 bars in tension, the results are:

```
>> checkshear(1693E3,16,23,350)
Vumax =     7368000
Vuc =   7.1089e+05
Vumin =   1.2319e+06
Vusmin =   1.7077e+06
s_vusmin =   69.2136
s =   58.3333
Shear reinforcement is required. Provide 7-N12 bars at
58 mm spacings
```

This means that $(1693000 = V^{*}) > (\phi V_{u.\min} = 862330)$ and 7-N12 ligatures will be provided at 58 mm spacing for an extent of D=350 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 23-N16 in tension, with cover=35 mm and k1=1.3 is $L_{sy.tb} = 603$ mm. This means that each leg of this L bar this bar has to be at least 603 mm long.

Flexure and shear analysis for bottom of the leg

The design moment at the bottom third of the leg is M*=197.15 kNm (seeFigure 4-16). By running `flexanalysis.m` for 15-N12 bars in tension, the results are:

```
>> flexanalysis(350,15,12,0,12) ;
dn =   11.5600
ku =    0.0374
Cc =  825.3840
Cs =      0
Mu =  251.7042
```

```
phiMu =  201.3633

Number of N12 bars required for compression:  0
Number of N12 bars required for tension: 15
```

This means that:

- the section is under-reinforced since $k_u<0.36$;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the bottom of the leg is V*=571.5 kN (see Figure 4-17). By running checkshear.m for the 15-N12 bars in tension, the results are:

```
>> checkshear(571.5E3,12,15,350)
Vumax =     7416000
Vuc =   5.0650e+05
Vumin =   1.0309e+06
Asvmin =  356.3818
ans =Minimum shear reinforcement is required.
```

This means that $0.5\phi V_{uc} < V^* \leq \phi V_{u.\min}$ and $A_{sv.\min}$ will be provided for an extent of D=350 mm. In this case, $V^* > \phi V_{uc}$ so the minimum shear reinforcement requirements cannot be waived. Using the maximum spacing of s=0.5D=175 mm from AS3600-2009 clause 8.2.12.2 into Equation 3-4, the result is $A_{sv.\min} = 356.3818$ mm, which means there will be 4-N12 ligatures required at 116 mm spacings.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 15-N12 bars in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 348$ mm. This means this bar has to be at least 696 mm long.

The final 1818 RCBC reinforcement is shown diagrammatically in Figure 4-18.

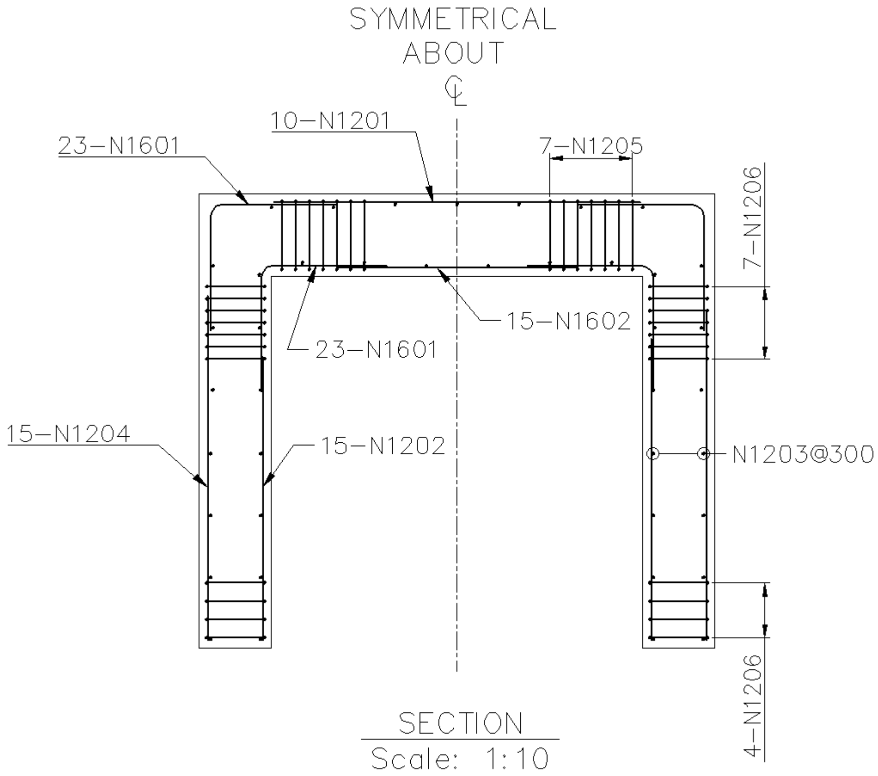

Figure 4-18 - 1818 RCBC reinforcement

In order to simplify the assembly of the reinforcement cages, especially with relation to the ligatures, the L bars at the end of the crown / top of the leg (N1601) were made the same length with the same spacing. The bars N1201 and N1204 were included to enable the connection of the distribution bars for cracking and the shear ligatures.

The reinforcement schedule for the 1818 RCBC is shown in Table 4-2.

Table 4-3 - 1818 RCBC reinforcement schedule

Bar Mark	Grade & Size	Qty	Total mass (kg)	Shape
N1201	N12	10	15.77	straight
N1202	N12	30	36.63	straight
N1203	N12	41	84.84	straight
N1204	N12	30	44.49	straight
N1205	N12	70	113.82	Ligs
N1206	N12	110	169.28	Ligs
N1601	N16	92	174.00	L
N1602	N16	15	27.50	straight
	Total reinforcement mass:		663.33	kg

4.6 2412 RCBC Design

To design the RCBC with 2.4 m span and 1.2 m leg (2412 RCBC), the cross section utilised comprised of a 350 mm leg and 400 mm crown. The overall culvert width was therefore 3.1 m and the overall height was 1.6 m.

The results from the load combination Matlab script `finalscript.m` (see appendix D for code) were as shown in Figure 4-19 and Figure 4-20. There are 20 straight lines in Figure 4-19, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 650.2 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 105.8 kPa.

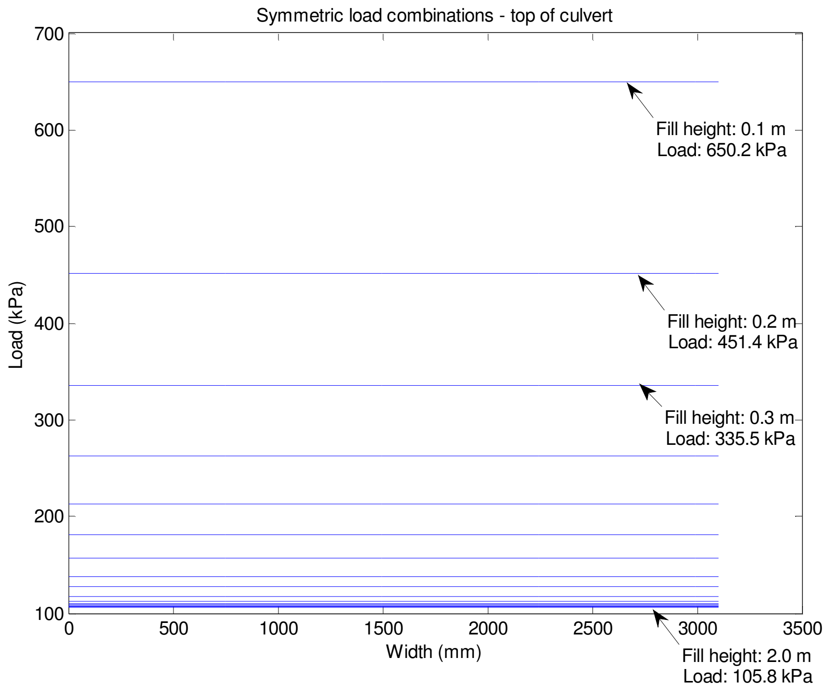

Figure 4-19 – Loads on top of RCBC 2412

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors. The horizontal loads combinations were the same as for the 1818 RCBC, since the loads on the culvert are the same (SM1600) and the culvert height is the same. They are shown in Figure 4-20.

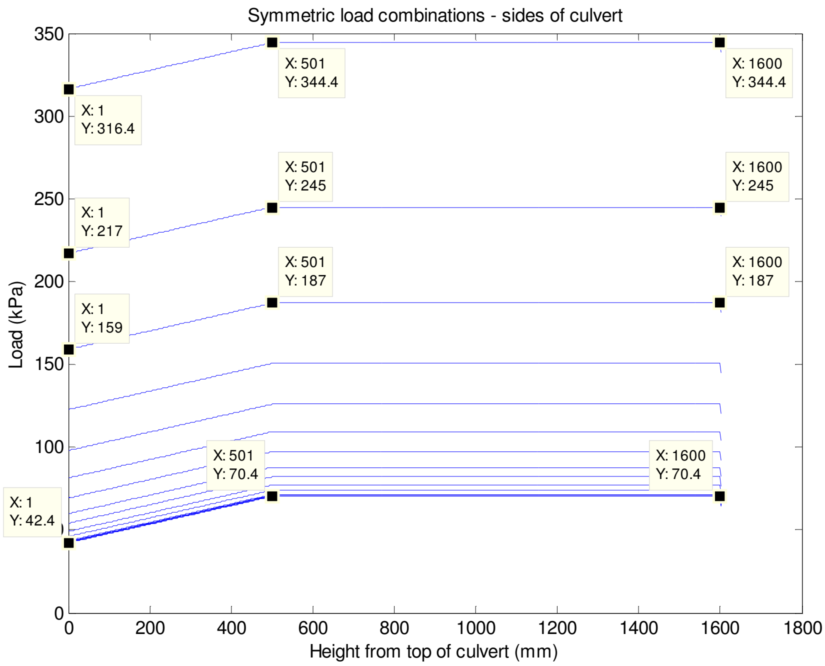

Figure 4-20 - Loads on the side of the 2412 RCBC

The 2412 RCBC with crown thickness of 400 mm and the leg thickness of 350 mm is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-21 and Figure 4-22.

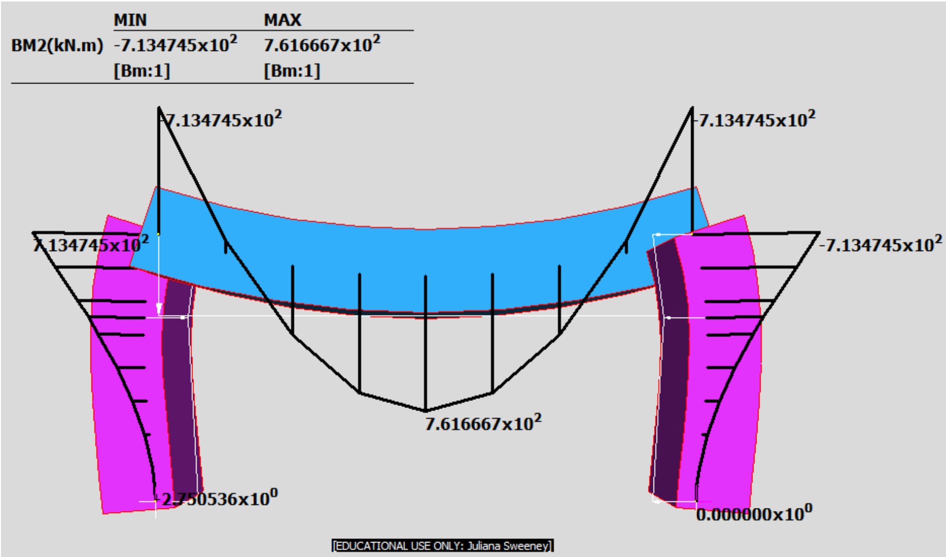

Figure 4-21 - Bending moment diagram for 2412 RCBC

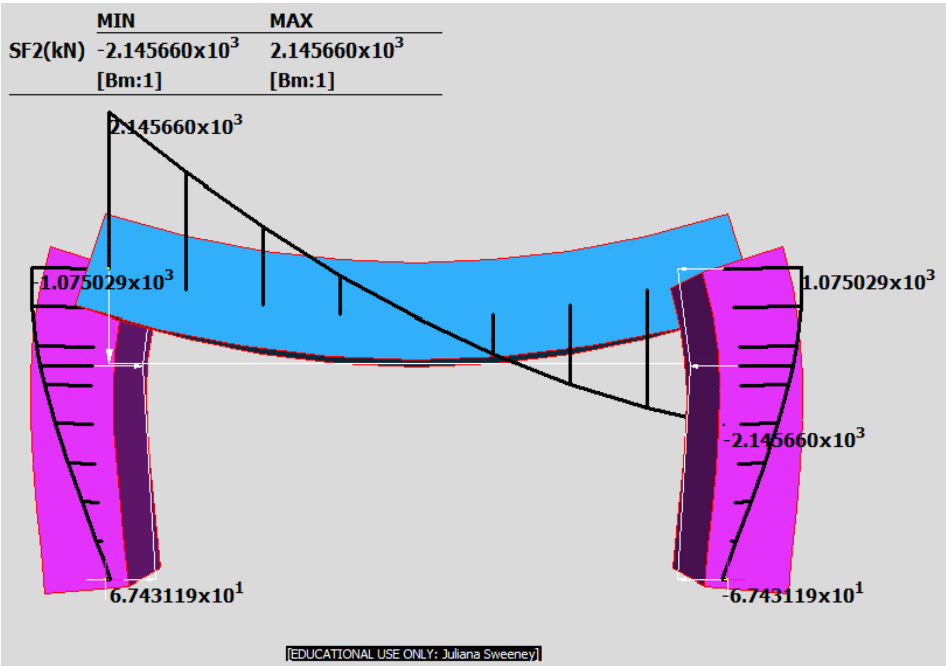

Figure 4-22 - Shear force diagram for 2412 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in detail in Section 3.3.5 - Non-optimised Culverts and Section 3.3.6 - Reinforcement.

Flexure and shear analysis for the middle of the crown

For the middle of the crown, M*=761.7 kNm (Figure 4-21) and by running `flexanalysis.m` with 19-N20 bars as tensile reinforcement, the results are:

```
>> flexanalysis(400,19,20,0,12);
dn =   41.2800
ku =    0.1163
Cc =   2.9474e+03
Cs =     0
Mu =  1.0037e+03
phiMu =  802.9922
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 19
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 19-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means this bar has to be at least 1060 mm long.

The shear for the middle of the crown is 0 so a shear check does not need to be carried out and no shear reinforcement is required.

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg.

Flexure and shear analysis for end of the crown

The design moment at the end of the crown is M*=713.5 kNm (see Figure 4-21). By running `flexanalysis.m` for 18-N20 bars in tension, the results are:

```
>> flexanalysis(400,18,20,0,12);
dn =   39.1100
ku =    0.1102
Cc =   2.7925e+03
Cs =     0
Mu =  953.0967
phiMu =  762.4773
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 18
```

This means that:

- the section is under-reinforced since $k_u<0.36$;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The design shear force at the end of the crown is V*=2145.7 kN (see Figure 4-22). By running `checkshear.m` for the 18-N20 bars in tension, the results are:

```
>> checkshear(2145.7E3,20,18,400);
Vumax =     8520000
Vuc =   8.0425e+05
Vumin =   1.4067e+06
Vusmin =   2.2610e+06
s_vusmin =   69.0833
s =   57.1429
Shear reinforcement is required. Provide 8-N12 bars at
57 mm spacings
```

This means that $(2145700 = V^*) > (\phi V_{u.\min} = 984690)$ and 8-N12 ligatures will be provided at 57 mm spacing for an extent of D=400 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 18-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means that each leg of this L bar this bar has to be at least 580 mm long.

<u>Flexure and shear analysis for top of the leg</u>

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=713.5 kNm (see Figure 4-21). However, this section is thinner since the leg is 350 mm and the crown is 400 mm. By running `flexanalysis.m` for 22-N20 bars in tension, the results are:

```
>>flexanalysis(350,22,20,0,12) ;
dn =   47.8000
ku =    0.1567
Cc =   3.4129e+03
Cs =     0
Mu =  983.8424
phiMu =  787.0740
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 22
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=1075 kN (see Figure 4-22). By running checkshear.m for the 22-N20 bars in tension, the results are:

```
>> checkshear(1183E3,20,22,350)
Vumax =      7320000
Vuc =   8.0833e+05
Vumin =   1.3259e+06
Vusmin =   7.2739e+05
s_vusmin =  115.3102
s =   87.5000
Shear reinforcement is required. Provide 5-N12 bars at
88 mm spacings
```

This means that $(1075000 = V^{*}) > (\phi V_{u.\min} = 928130)$ and 5-N12 ligatures will be provided at 88 mm spacing for an extent of D=350 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 22-N20 in tension, with cover=35 mm and k1=1.3 is $L_{sy.tb} = 754$ mm. This means that each leg of this L bar this bar has to be at least 754 mm long.

<u>Flexure and shear analysis for bottom of the leg</u>

The design moment at 0.5 m from the bottom of the leg is M*=69.9 kNm (see Figure 4-23).

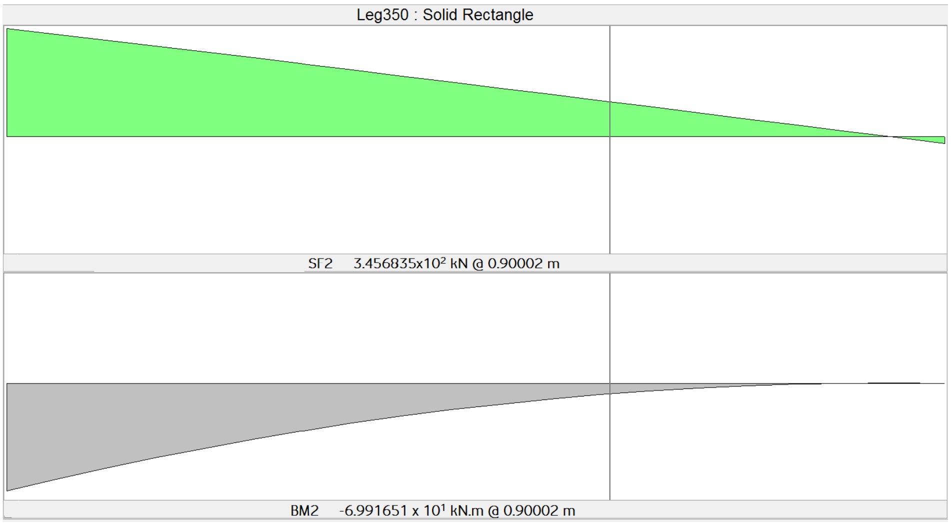

Figure 4-23 - Bending moment and shear force for the 2412 RCBC leg

By running `flexanalysis.m` for 6-N12 bars in tension, the results are:

```
>> flexanalysis(350,6,12,0,12) ;
dn =    4.6200
ku =    0.0150
Cc =  329.8680
Cs =      0
Mu =  101.3958
phiMu =   81.1167
Number of N12 bars required for compression:  0
Number of N12 bars required for tension:  6
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. At 0.5 m from the bottom of the leg is V*=346 kN (see Figure 4-23). By running `checkshear.m` for the 6-N12 bars in tension, the results are:

```
>> checkshear(345E3,12,6,350)
Vumax =      7416000
Vuc =   3.7320e+05
Vumin =   8.9759e+05
Asvmin =  356.3818
ans = Minimum shear reinforcement is required.
```

This means that $0.5\phi V_{uc} < V^* \le \phi V_{u.\min}$ and $A_{sv.\min}$ will be provided for an extent of D=350 mm. In this case, $V^* > \phi V_{uc}$ so the minimum shear reinforcement requirements cannot be waived. However, if the number of N12 bars is increased to 14-N12 in tension, the results from `flexanalysis.m` are:

```
>>flexanalysis(350,14,12,0,12)
dn =   10.7900
ku =    0.0349
Cc =  770.4060
Cs =     0
Mu =  235.1460
phiMu =  188.1168
Number of N12 bars required for compression:  0
Number of N12 bars required for tension: 14
```

That means this reinforcement would also be suitable, but in this case, the shear reinforcement requirements could be waived. This is shown by running `checkshear.m`:

```
checkshear(345.7E3,12,14,350)
Vumax =      7416000
Vuc =   4.9499e+05
Vumin =   1.0194e+06
ans = Minimum shear reinforcement is required but may
be waived.
```

In this case, $V^* \le \phi V_{uc}$ so the minimum shear reinforcement requirements can be waived. It is more feasible to provide the extra 8-N12 straight bars than it is to provide the required amount of shear reinforcement. The 32 ligatures required in total weigh approximately 50 kg while the extra 16-N12 bars required in total weigh approximately 22 kg. Also, it simplifies the assembly of the cage since it is simpler to install straight bars than ligatures.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 14-N12 bars in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 348$ mm. This means this bar has to be at least 696 mm long.

The final 2412 RCBC reinforcement is shown diagrammatically in Figure 4-24.

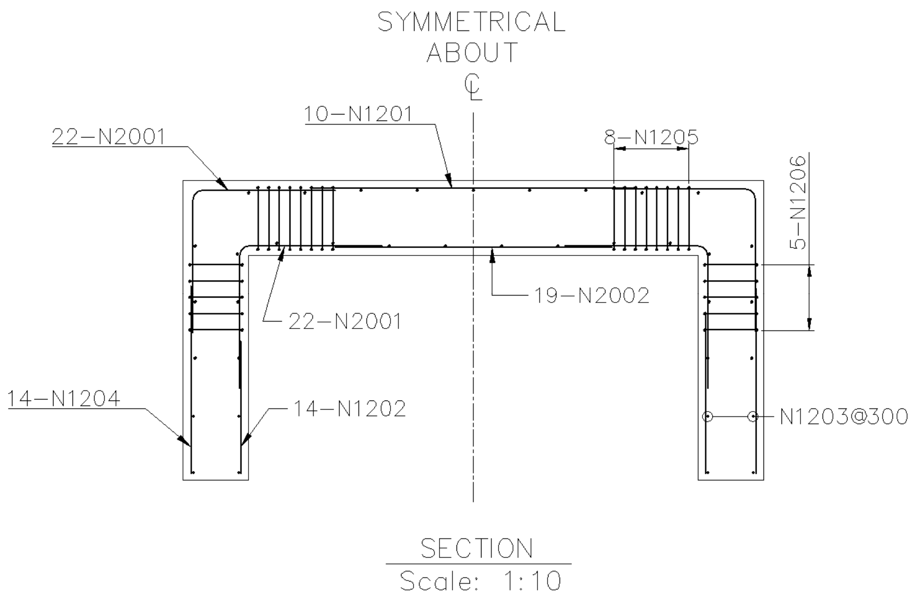

Figure 4-24 - 2412 RCBC reinforcement

In order to simplify the assembly of the reinforcement cages, especially with relation to the ligatures, the L bars at the end of the crown / top of the leg (N2001) were made the same length with the same spacing. The bars N1201 and N1204 were included to enable the connection of the distribution bars for cracking. The reinforcement schedule for the 2412 RCBC is shown in Table 4-4.

Table 4-4 - 2412 RCBC reinforcement schedule

Bar Mark	Grade & Size	Qty	Total mass (kg)	Shape
N1201	N12	10	15.37	straight
N1202	N12	28	17.41	straight
N1203	N12	37	76.56	straight
N1204	N12	28	22.38	straight
N1205	N12	80	130.08	Ligs
N1206	N12	50	76.95	Ligs
N2001	N20	88	327.13	L
N2002	N20	19	59.61	straight
	Total reinforcement mass:		725.49	kg

4.7 2415 RCBC Design

To design the RCBC with 2.4 m span and 1.5 m leg (2415 RCBC), the cross section utilised comprised of a 350 mm leg and 400 mm crown. The overall culvert width was therefore 3.1 m and the overall height was 1.9 m.

The results from the load combination Matlab script `finalscript.m` (see appendix D for code) were as shown in Figure 4-30 and Figure 4-31. The loads on the top of the culvert are uniformly distributed over the entire top of culvert, as expected since the truncated prism model (see Section 3.2 for details) distributes the vehicle and construction loads uniformly over the top of the culvert. The other loads that act on top of the culvert are also uniformly distributed, namely the fill and the self-weight. There are 20 straight lines in Figure 4-30, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 651.9 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 107.5 kPa.

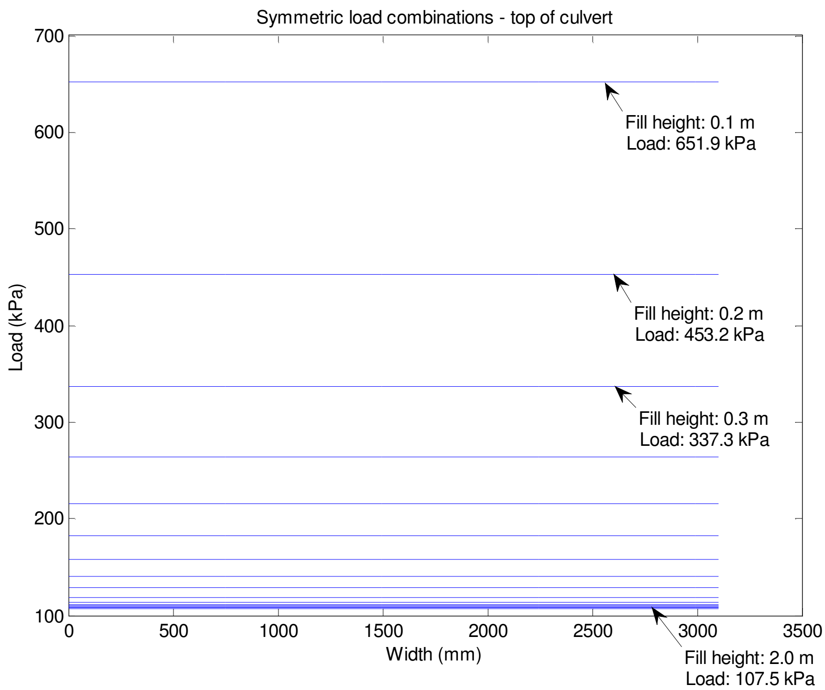

Figure 4-25 – Loads on top of 2415 RCBC

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors. The horizontal loads combinations were the same as for the 1815 RCBC, since the loads on the culvert are the same (SM1600) and the culvert height is the same. They are shown in Figure 4-31.

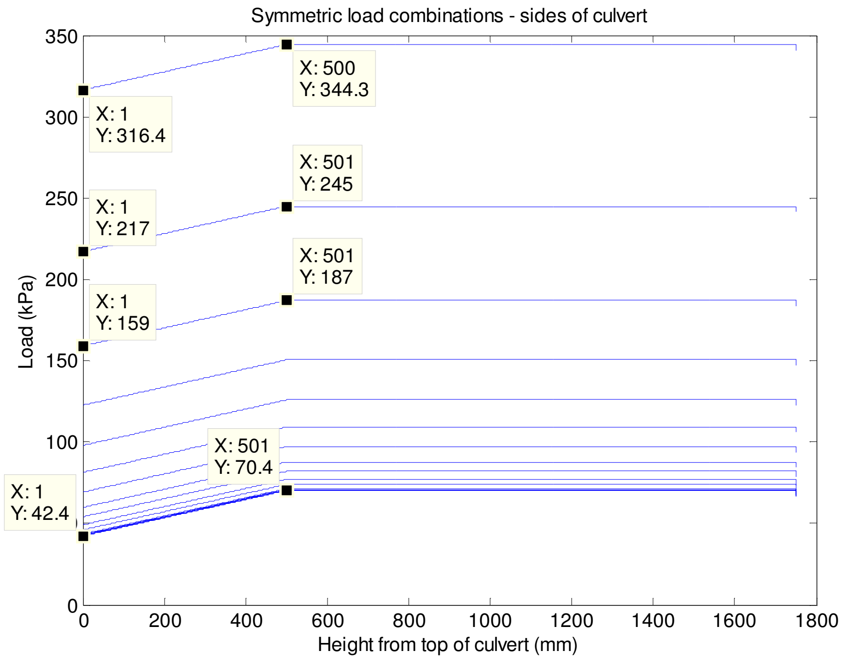

Figure 4-26 - Loads on the side of the 2415 RCBC

The 2415 RCBC with crown thickness of 400 mm and the leg thickness of 350 mm is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-32 and Figure 4-33.

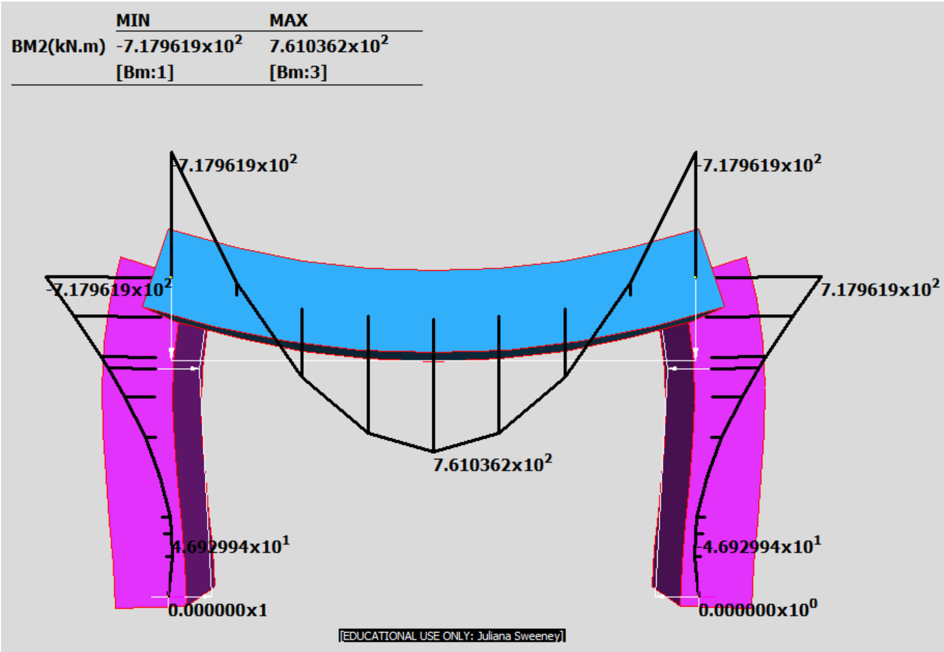

Figure 4-27 - Bending moment diagram for 2415 RCBC

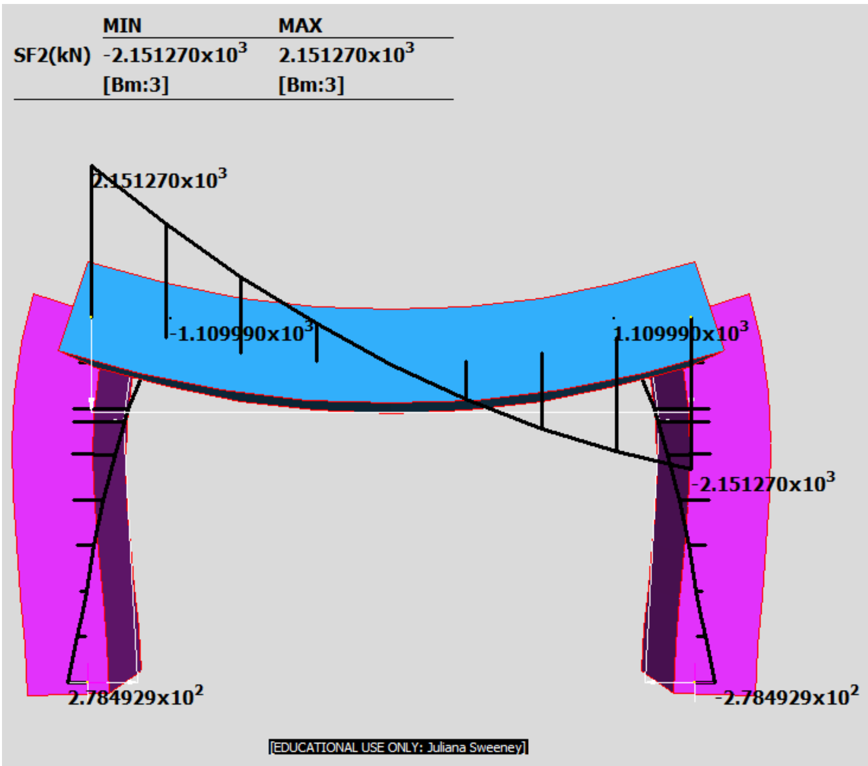

Figure 4-28 - Shear force diagram for 2415 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in detail in Section 3.3.5 - Non-optimised Culverts and Section 3.3.6 - Reinforcement.

Flexure and shear analysis for the middle of the crown

For the middle of the crown, M*=761.0 kNm (Figure 4-32) and by running `flexanalysis.m` with 19-N20 bars as tensile reinforcement, the results are:

```
>> flexanalysis(400,19,20,0,12);
dn =   41.2800
ku =    0.1163
Cc =   2.9474e+03
Cs =     0
Mu =   1.0037e+03
phiMu =  802.9922
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 19
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 19-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means this bar has to be at least 1060 mm long.

The shear for the middle of the crown is 0 so a shear check does not need to be carried out and no shear reinforcement is required.

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg.

Flexure and shear analysis for end of the crown

The design moment at the end of the crown is M*=718.0 kNm (see Figure 4-32). By running `flexanalysis.m` for 18-N20 bars in tension, the results are:

```
>> flexanalysis(400,18,20,0,12)
dn =   39.1100
ku =    0.1102
Cc =   2.7925e+03
Cs =     0
Mu =  953.0967
phiMu =  762.4773
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 18
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The design shear force at the end of the crown is V*=2151.2 kN (see Figure 4-33). By running `checkshear.m` for the 18-N20 bars in tension, the results are:

```
>> checkshear(2151.2E3,20,18,400);
Vumax =      8520000
Vuc =   8.0425e+05
Vumin =   1.4067e+06
Vusmin =   2.2689e+06
s_vusmin =   68.8441
s =   57.1429
```

```
Shear reinforcement is required. Provide 8-N12 bars at
57 mm spacings
```

This means that $(2151200 = V^{*}) > (\phi V_{u.\min} = 984690)$ and 8-N12 ligatures will be provided at 57 mm spacing for an extent of D=400 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 18-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means that each leg of this L bar this bar has to be at least 580 mm long.

<u>Flexure and shear analysis for top of the leg</u>

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=718.0 kNm (see Figure 4-32). However, this section is thinner since the leg is 350 mm and the crown is 400 mm. By running `flexanalysis.m` for 22-N20 bars in tension, the results are:

```
>>flexanalysis(350,22,20,0,12) ;
dn =   47.8000
ku =    0.1567
Cc =   3.4129e+03
Cs =      0
Mu =  983.8424
phiMu =  787.0740
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 22
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^{*}$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=1110 kN (seeFigure 4-33). By running `checkshear.m` for the 22-N20 bars in tension, the results are:

```
>> checkshear(1183E3,20,23,350)
Vumax =     7320000
Vuc =   8.0833e+05
Vumin =   1.3259e+06
Vusmin =   7.7739e+05
s_vusmin =  107.8937
s =   87.5000
Shear reinforcement is required. Provide 5-N12 bars at
88 mm spacings
```

This means that $(1110000 = V^{*}) > (\phi V_{u.\min} = 928130)$ and 5-N12 ligatures will be provided at 88 mm spacing for an extent of D=350 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 22-N20 in tension, with cover=35 mm and k1=1.3 is $L_{sy.tb} = 754$ mm. This means that each leg of this L bar this bar has to be at least 754 mm long.

Flexure and shear analysis for bottom of the leg

The maximum design moment at the bottom third of the leg is M*=46.9 kNm (see Figure 4-32). By running `flexanalysis.m` for 8-N12 bars in tension, the results are:

```
>> flexanalysis(350,8,12,0,12) ;
dn =    6.1600
ku =    0.0199
Cc =  439.8240
Cs =     0
Mu =  134.9574
```

```
phiMu =  107.9659
Number of N12 bars required for compression:  0
Number of N12 bars required for tension:  8
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The reason why this seems overdesigned is that if less bars than 8-N12s are utilised, the minimum shear reinforcement requirement cannot be waived. 8-N12 bars are then chosen to eliminate the need to provide shear reinforcement. The design shear force at the bottom of the leg is V*=278.5 kN (see Figure 4-33). By running `checkshear.m` for the 8-N12 bars in tension, the results are:

```
>> checkshear(278.5E3,12,8,350)
Vumax =     7416000
Vuc =   4.1076e+05
Vumin =   9.3515e+05
ans =Minimum shear reinforcement is required but may be
waived.
```

This means that $0.5\phi V_{uc} < V^* \leq \phi V_{u.\min}$ and $A_{sv.\min}$ should be provided for an extent of D=350 mm. In this case, $V^* \leq \phi V_{uc}$ so the minimum shear reinforcement requirements can be waived.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 8-N12 bars in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 348$ mm. This means this bar has to be at least 696 mm long.

The final 2415 RCBC reinforcement is shown diagrammatically in Figure 4-34.

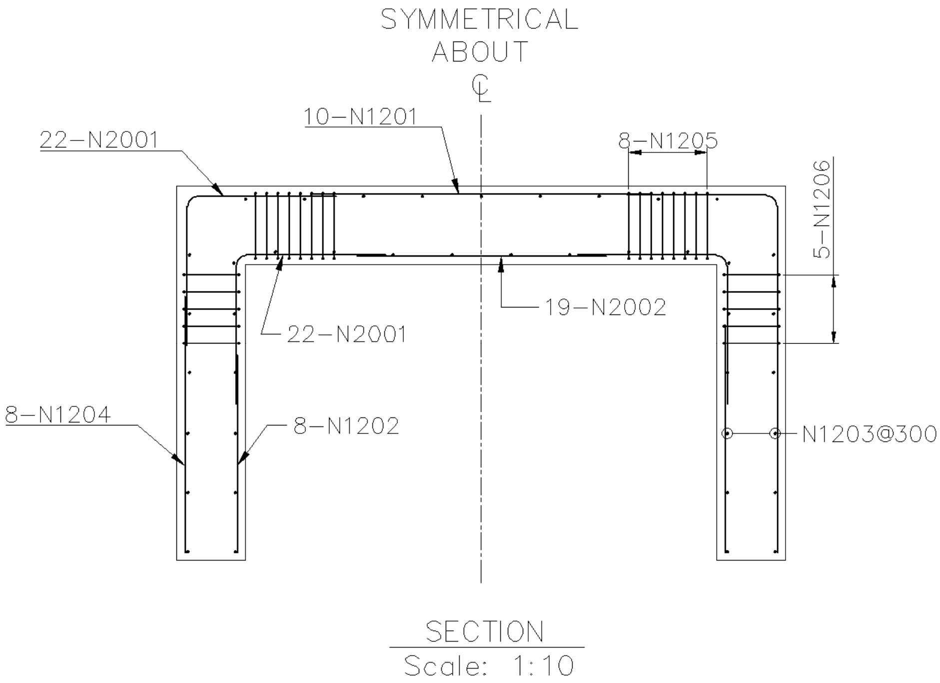

Figure 4-29 - 2415 RCBC reinforcement

In order to simplify the assembly of the reinforcement cages, especially with relation to the ligatures, the L bars at the end of the crown / top of the leg (N2001) were made the same length with the same spacing. The bars N1201 and N1204 were included to enable the connection of the distribution bars for cracking and the shear ligatures.

The reinforcement schedule for the 2415 RCBC is shown in Table 4-5.

Table 4-5 - 2415 RCBC reinforcement schedule

Bar Mark	Grade & Size	Qty	Total mass (kg)	Shape
N1201	N12	10	15.37	straight
N1202	N12	16	14.21	straight
N1203	N12	41	84.84	straight
N1204	N12	16	17.05	straight
N1205	N12	80	130.08	Ligs
N1206	N12	50	76.95	Ligs
N2001	N20	88	327.13	L
N2002	N20	19	59.61	straight
	Total reinforcement mass:		725.24	kg

4.8 2418 RCBC Design

To design the RCBC with 2.4 m span and 1.8 m leg (2418 RCBC), the cross section utilised comprised of a 350 mm leg and 400 mm crown. The overall culvert width was therefore 3.1 m and the overall height was 2.2 m.

The results from the load combination Matlab script `finalscript.m` (see appendix D for code) were as shown in Figure 4-30 and Figure 4-31. The loads on the top of the culvert are uniformly distributed over the entire top of culvert, as expected since the truncated prism model (see Section 3.2 for details) distributes the vehicle and construction loads uniformly over the top of the culvert. The other loads that act on top of the culvert are also uniformly distributed, namely the fill and the self-weight. There are 20 straight lines in Figure 4-30, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 653.7 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 109.3 kPa.

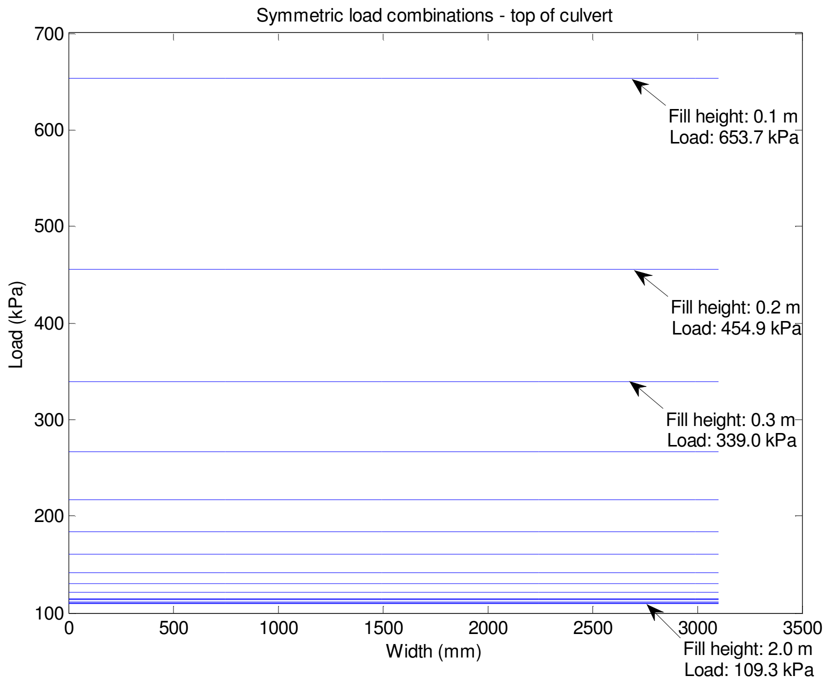

Figure 4-30 – Loads on top of 2418 RCBC

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors. The horizontal loads combinations were the same as for the 1818 RCBC, since the loads on the culvert are the same (SM1600) and the culvert height is the same. They are shown in Figure 4-31.

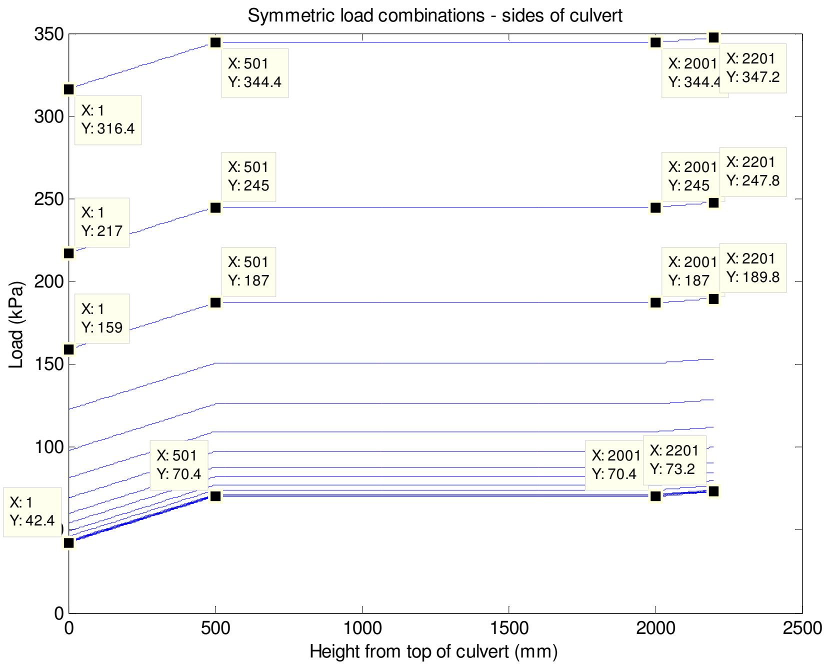

Figure 4-31 - Loads on the side of the 2418 RCBC

The 2418 RCBC with crown thickness of 400 mm and the leg thickness of 350 mm is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-32 and Figure 4-33.

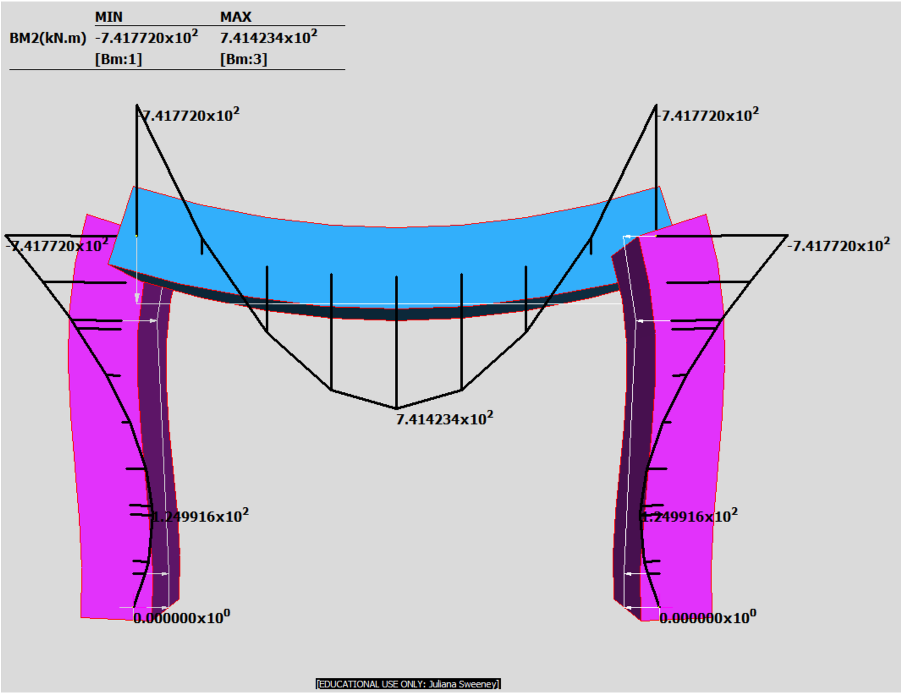

Figure 4-32 - Bending moment diagram for 2418 RCBC

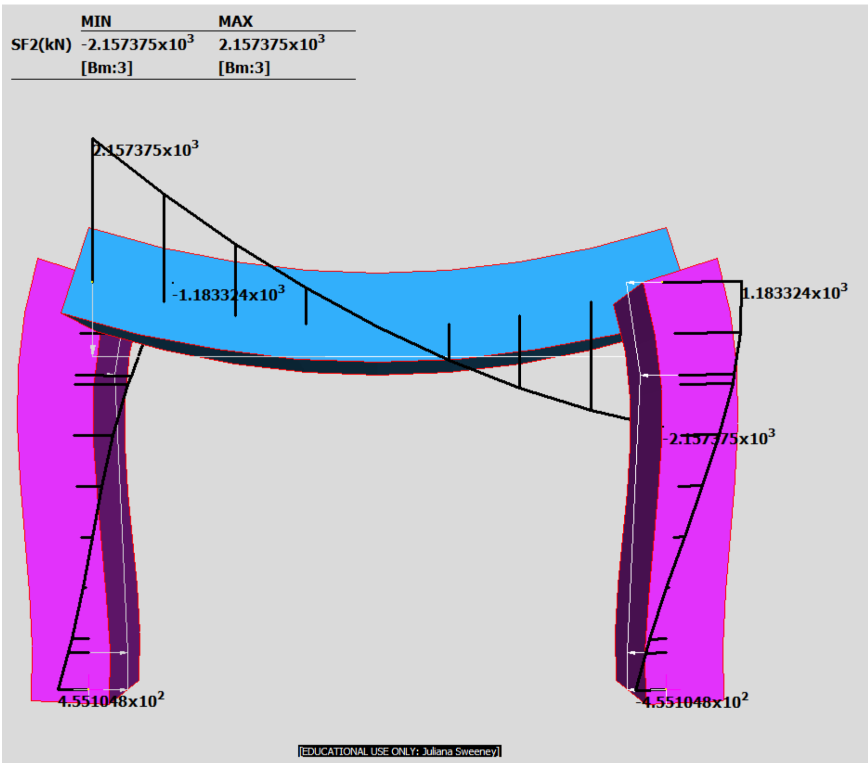

Figure 4-33 - Shear force diagram for 2418 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in detail in Section 3.3.5 - Non-optimised Culverts and Section 3.3.6 - Reinforcement.

Flexure and shear analysis for the middle of the crown

For the middle of the crown, M*=741.4 kNm (Figure 4-32) and by running `flexanalysis.m` with 19-N20 bars as tensile reinforcement, the results are:

```
>> flexanalysis(400,19,20,0,12);
dn =   41.2800
ku =    0.1163
Cc =   2.9474e+03
Cs =      0
Mu =   1.0037e+03
phiMu =  802.9922
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 19
```

This means that:

- the section is under-reinforced since $k_u<0.36$;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 19-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means this bar has to be at least 1060 mm long.

The shear for the middle of the crown is 0 so a shear check does not need to be carried out and no shear reinforcement is required.

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg.

Flexure and shear analysis for end of the crown

The design moment at the end of the crown is M*=741.8 kNm (see Figure 4-32). By running `flexanalysis.m` for 19-N20 bars in tension, the results are:

```
>> flexanalysis(400,19,20,0,12)
dn =   41.2800
ku =    0.1163
Cc =   2.9474e+03
Cs =     0
Mu =   1.0037e+03
phiMu =  802.9922
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 19
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The design shear force at the end of the crown is V*=2157.4 kN (see Figure 4-33). By running `checkshear.m` for the 19-N20 bars in tension, the results are:

```
>> checkshear(2157.4E3,20,19,400);
Vumax =     8520000
Vuc =   8.1887e+05
Vumin =   1.4213e+06
Vusmin =   2.2631e+06
s_vusmin =   69.0196
s =   57.1429
```

```
Shear reinforcement is required. Provide 8-N12 bars at
57 mm spacings
```

This means that $(2157400 = V^*) > (\phi V_{u.\min} = 994910)$ and 8-N12 ligatures will be provided at 57 mm spacing for an extent of D=400 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 19-N20 in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 580$ mm. This means that each leg of this L bar this bar has to be at least 580 mm long.

<u>Flexure and shear analysis for top of the leg</u>

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=741.8 kNm (seeFigure 4-32). However, this section is thinner since the leg is 350 mm and the crown is 400 mm. By running `flexanalysis.m` for 23-N20 bars in tension, the results are:

```
>>flexanalysis(350,23,20,0,12) ;
dn =   49.9700
ku =    0.1638
Cc =   3.5679e+03
Cs =     0
Mu =   1.0258e+03
phiMu =  820.6373
Number of N12 bars required for compression:  0
Number of N20 bars required for tension: 23
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=1183 kN (see Figure 4-33). By running `checkshear.m` for the 23-N20 bars in tension, the results are:

```
>> checkshear(1183E3,20,23,350)
Vumax =      7320000
Vuc =   8.2040e+05
Vumin =   1.3380e+06
Vusmin =   8.6960e+05
s_vusmin =   96.4518
s =   87.5000
Shear reinforcement is required. Provide 5-N12 bars at
88 mm spacings
```

This means that $(1183000 = V^{*}) > (\phi V_{u.\min} = 936600)$ and 5-N12 ligatures will be provided at 88 mm spacing for an extent of D=350 mm.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 23-N20 in tension, with cover=35 mm and k1=1.3 is $L_{sy.tb} = 754$ mm. This means that each leg of this L bar this bar has to be at least 754 mm long.

Flexure and shear analysis for bottom of the leg

The design moment at the bottom third of the leg is M*=125 kNm (seeFigure 4-32). By running `flexanalysis.m` for 10-N12 bars in tension, the results are:

```
>> flexanalysis(350,10,12,0,12) ;
dn =    7.7100
ku =    0.0250
Cc =  550.4940
Cs =      0
Mu =  168.6171
```

```
phiMu =  134.8937
Number of N12 bars required for compression:  0
Number of N12 bars required for tension: 10
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the bottom of the leg is V*=455.1 kN (seeFigure 4-33). By running `checkshear.m` for the 10-N12 bars in tension, the results are:

```
>> checkshear(455.1E3,12,10,350)
Vumax =      7416000
Vuc =   4.4247e+05
Vumin =   9.6686e+05
Asvmin =  356.3818
ans =Minimum shear reinforcement is required.
```

This means that $0.5\phi V_{uc} < V^* \leq \phi V_{u.\min}$ and $A_{sv.\min}$ will be provided for an extent of D=350 mm. In this case, $V^* > \phi V_{uc}$ so the minimum shear reinforcement requirements cannot be waived. Using the maximum spacing of s=0.5D=175 mm from AS3600-2009 clause 8.2.12.2 into Equation 3-4, the result is $A_{sv.\min} = 356.3818$ mm, which means there will be 4-N12 ligatures required at 116 mm spacings.

The development length of the tension bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6. The result for 10-N12 bars in tension, with cover=35 mm and k1=1 is $L_{sy.tb} = 348$ mm. This means this bar has to be at least 696 mm long.

The final 2418 RCBC reinforcement is shown diagrammatically in Figure 4-34.

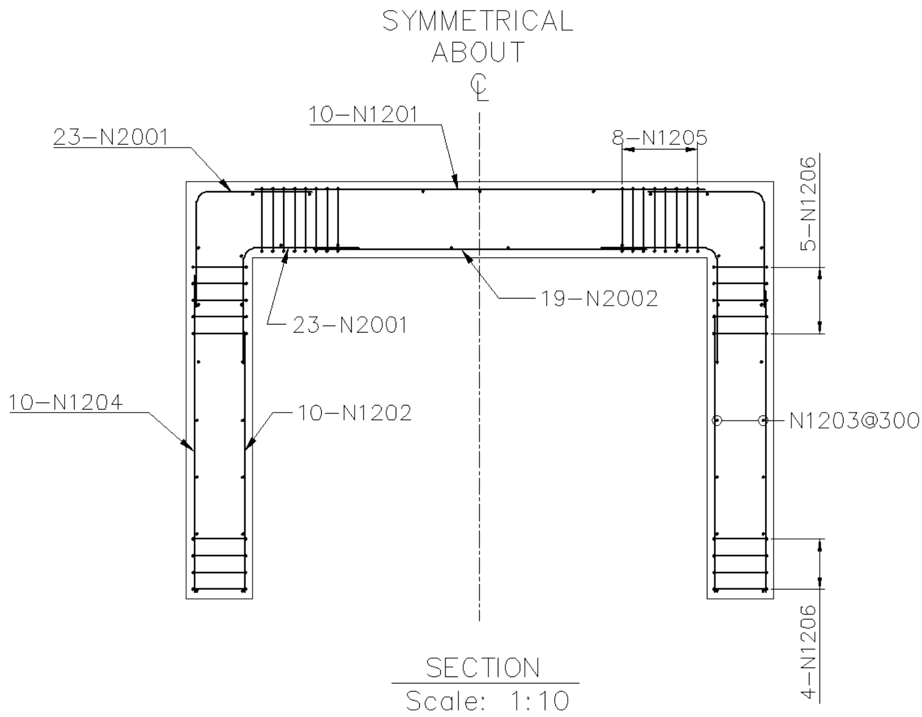

Figure 4-34 - 2418 RCBC reinforcement

In order to simplify the assembly of the reinforcement cages, especially with relation to the ligatures, the L bars at the end of the crown / top of the leg (N2001) were made the same length with the same spacing. The bars N1201 and N1204 were included to enable the connection of the distribution bars for cracking and the shear ligatures.

The reinforcement schedule for the 2418 RCBC is shown in Table 4-6.

Table 4-6 - 2418 RCBC reinforcement schedule

Bar Mark	Grade & Size	Qty	Total mass (kg)	Shape
N1201	N12	10	15.37	straight
N1202	N12	20	23.18	straight
N1203	N12	45	93.11	straight
N1204	N12	20	28.42	straight
N1205	N12	80	130.08	Ligs
N1206	N12	90	138.51	Ligs
N2001	N20	92	342.00	L
N2002	N20	19	69.46	straight
	Total reinforcement mass:		840.13	kg

4.9 Conclusion

The iterative process of designing the non-optimised RCBC to find its reinforcement layout can be time-consuming if done by hand or with primitive software, like in the case of this study. Manufacturers in the industry have access to a great variety of more advanced software that can make this procedure easy and quick, since all parameters are dictated by Australian Standards and Main Roads Standards.

An interesting result from this section is the fact that the fill height makes a great difference in how much load the box culvert is subjected to. There were great differences in the load supported by the RCBCs depending on the fill height, especially between 0.1m and 0.4m of fill. This means that the greater the fill height in real life installations, the more conservative the culvert design will have been, which can dramatically increase the factor of safety when utilising these structures.

In summary, due to the fact that different designers can come up with different compliant structures, the optimised culverts presented in the next section could end up very different, even though they would all comply with specifications and be fit for purpose. It would be valid to investigate manners to optimise the reinforcement, but it is not within the scope of this project.

CHAPTER 5 - TOPOLOGY OPTIMISATION

5.1 Introduction

The topology optimisation procedure was implemented by using a Matlab program called `top_rcbc4.m`, (see Appendix F for code) which was adapted from Sigmund (2001). The modifications made to the program as well as the explanation of what each part of the program does can be found in Section 3.4.1.

5.2 1815 RCBC

The topology optimisation Matlab program is applied to the 1815 RCBC with the following input parameters:

span in mm = 1800
leg height in mm = 1500
volume constraint = 0.25
penalisation factor = 3
filter size divided by the element size (r_{min}) = 1.2

The command `top_rcbc4(1800,1500,0.25,3,1.2)` yields the result shown in Figure 5-1. The simulation comprised of 135 iterations and took 138 seconds.

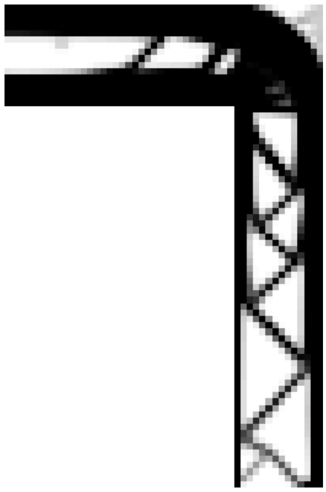

Figure 5-1 - Topology optimisation result for 1815 RCBC

The voids in the optimised culvert are to be created only where the colour map shows white, that is, where the relative density is 0.001. Grey areas mean composite materials and since this study is not going to utilise composite materials, only the white parts can become voids. The culvert would look like Figure 5-2.

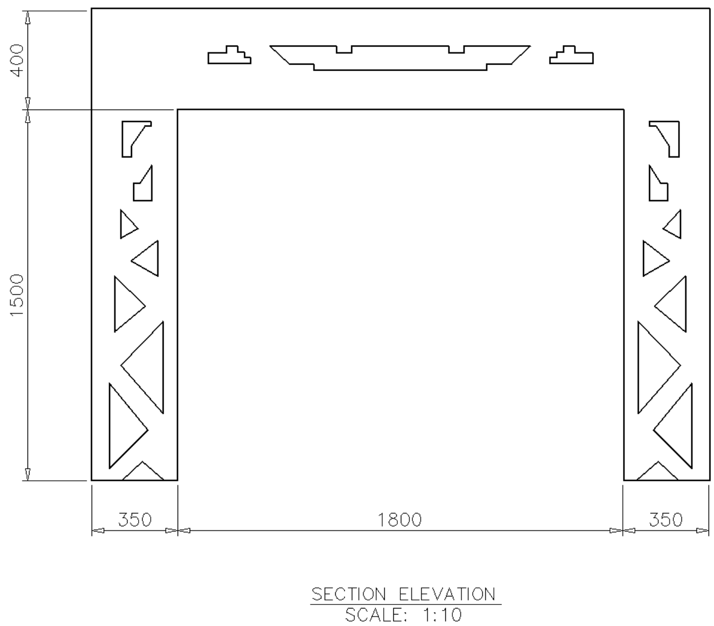

Figure 5-2 – Optimised 1815 RCBC section elevation

However, it can be seen the voids are at times too close to the edge of the culvert. We know the leg reinforcement for the 1815 RCBC leg is comprised of N16 bars with N12 distribution bars. That means the distance between the edge of the void to the edge of the leg needs to be a minimum of 35+35+16+12=98 mm, which is shown by the dashed line in Figure 5-3.

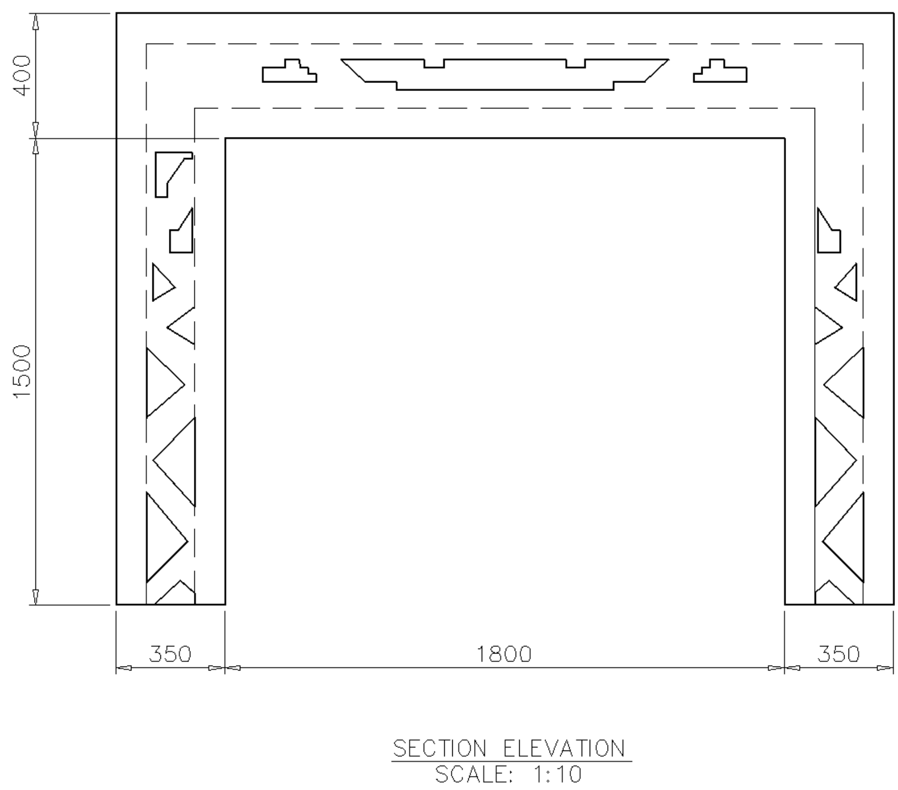

Figure 5-3 - Optimised 1815 with trimmed voids

The voids then need to be trimmed to be within the dashed line. Also, the reinforcement has to fit in and there needs to be 35 mm cover at all times. The 1815 RCBC with trimmed voids is shown in Figure 5-4 with superimposed reinforcement. It can be seen that some voids will not be possible because they coincide with the shear ligatures.

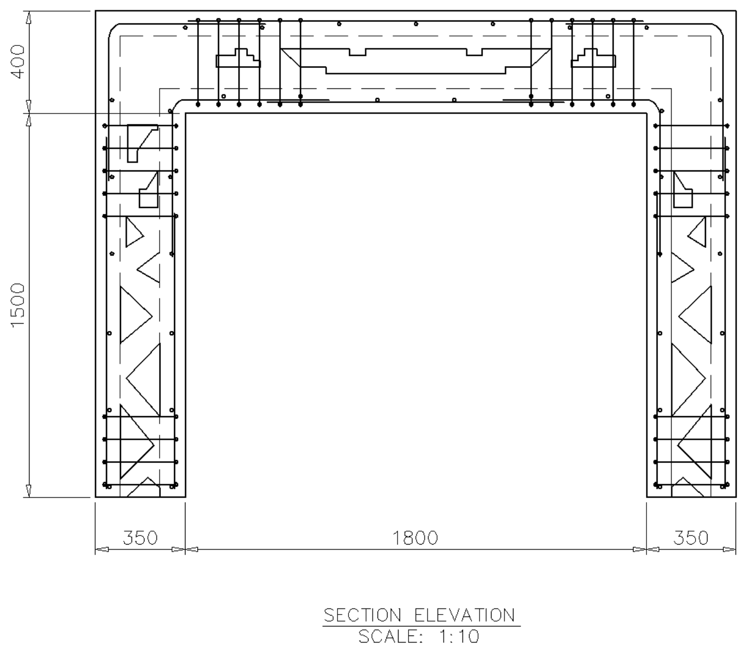

Figure 5-4 - Optimised 1815 with trimmed voids and reinforcement

If the impossible voids are removed, the final optimised culvert would look like shown in Figure 5-5.

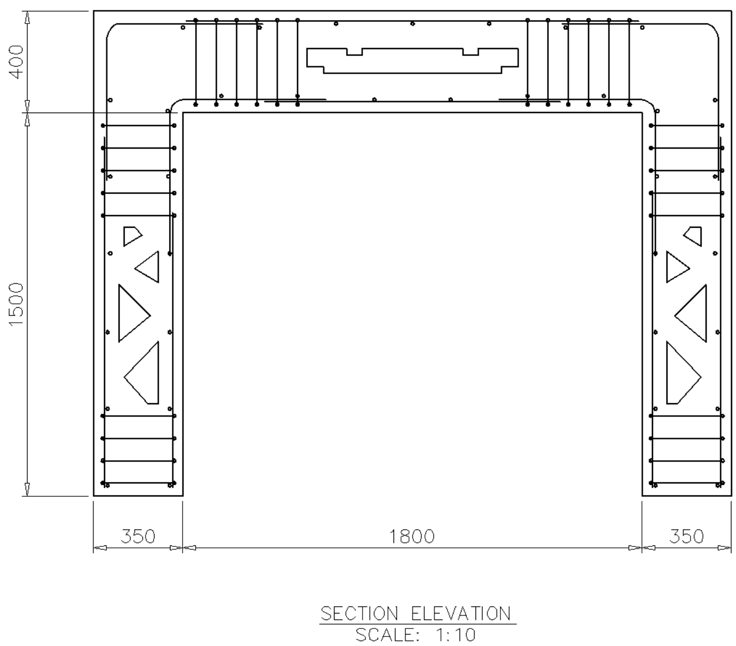

Figure 5-5 - Final 1815 optimised RCBC

5.3 1818 RCBC

The topology optimisation Matlab program is applied to the 1818 RCBC with the following input parameters:

span in mm = 1800

leg height in mm = 1800

volume constraint = 0.25

penalisation factor = 3

filter size divided by the element size (r_{min}) = 1.2

The command `top_rcbc4(1800,1800,0.25,3,1.2)` yields the result shown in Figure 5-11. The simulation comprised of 135 iterations and took 406 seconds.

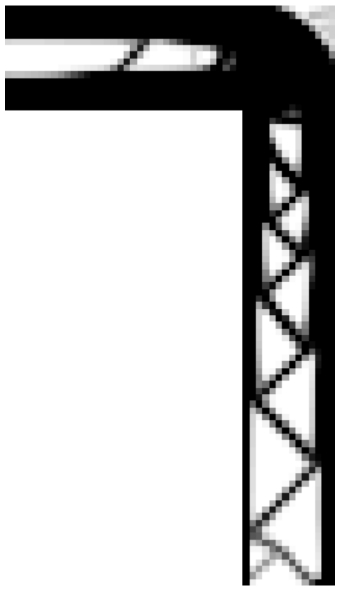

Figure 5-6 - Topology optimisation result for 1818 RCBC

The voids in the optimised culvert are to be created only where the colour map shows white, that is, where the relative density is 0.001. Grey areas mean composite materials and since this study is not going to utilise composite

materials, only the white parts can become voids. The culvert would look like Figure 5-12.

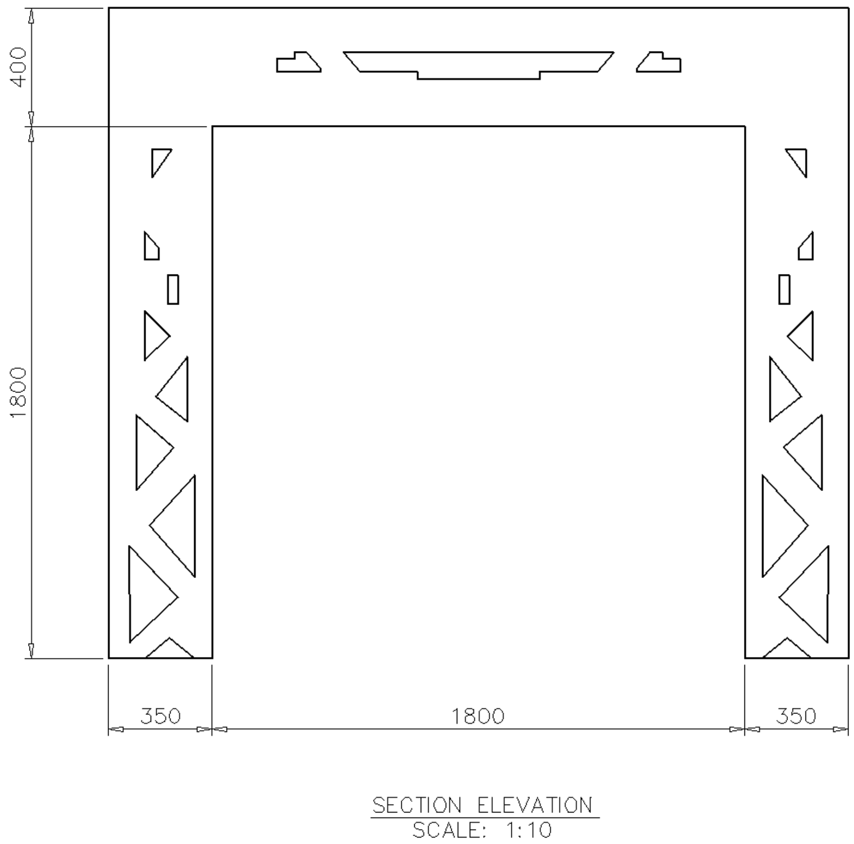

Figure 5-7 – Optimised 1815 RCBC section elevation

However, it can be seen the voids are at times too close to the edge of the culvert. We know the leg reinforcement for the 1815 RCBC leg is comprised of N16 bars with N12 distribution bars. That means the distance between the edge of the void to the edge of the leg needs to be a minimum of 35+35+16+12=98 mm, which is shown by the dashed line in Figure 5-3.

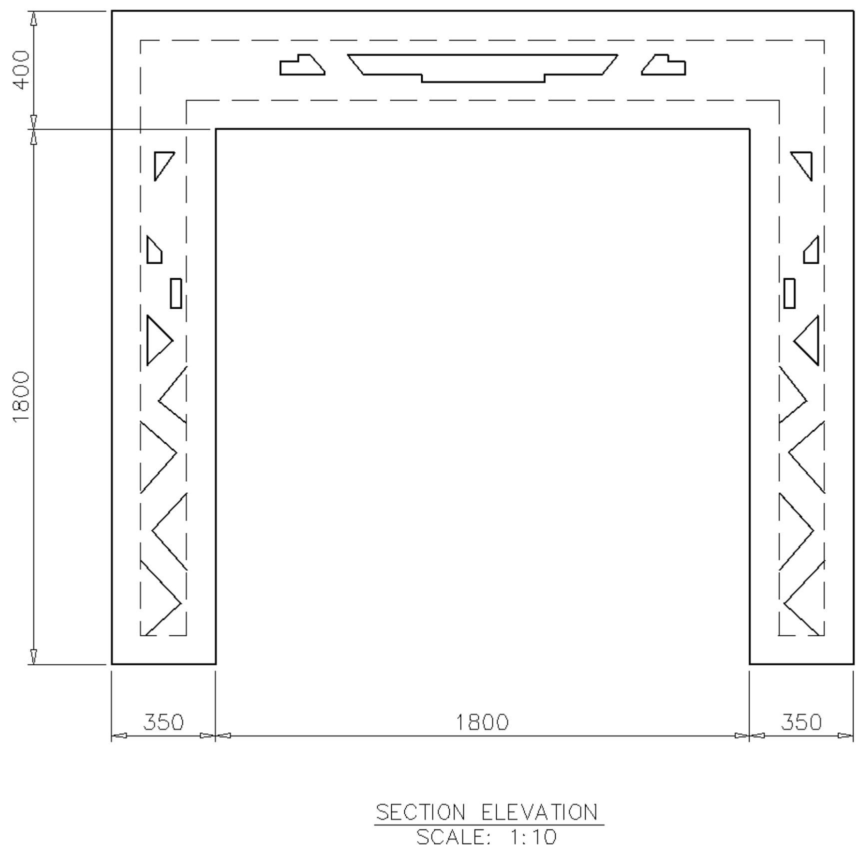

Figure 5-8 - Optimised 1815 RCBC with trimmed voids

The voids then need to be trimmed to be within the dashed line. Also, the reinforcement has to fit in and there needs to be 35 mm cover at all times. The 1818 RCBC with trimmed voids is shown in Figure 5-14 with superimposed reinforcement. It can be seen that some voids will not be possible because they coincide with the shear ligatures.

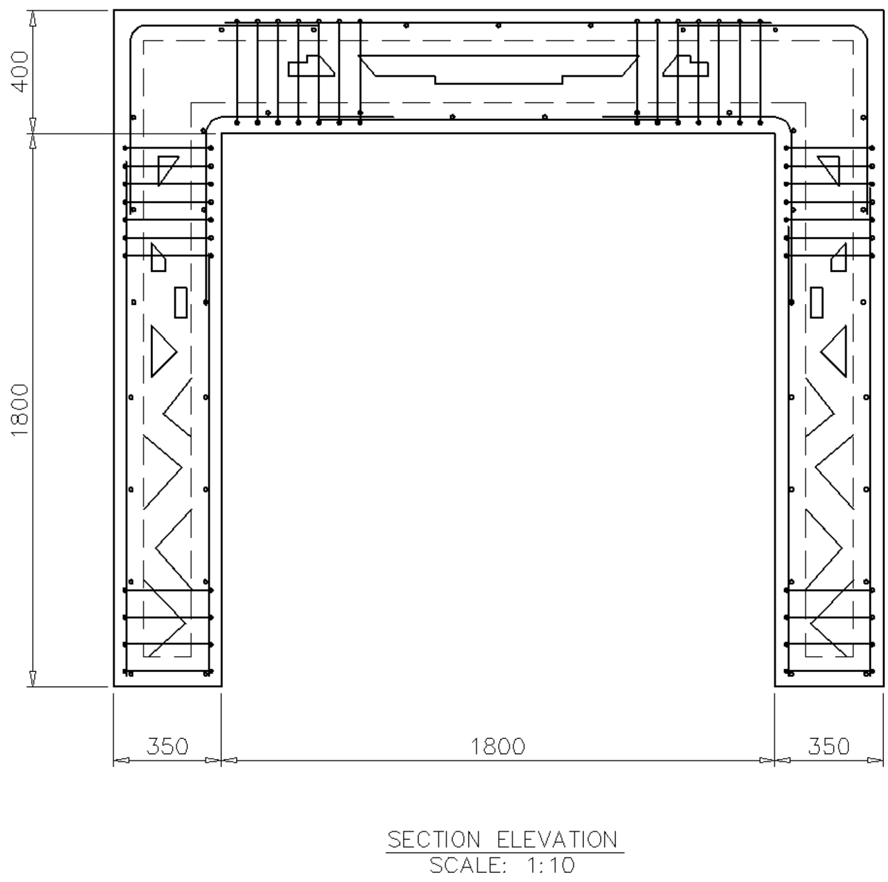

Figure 5-9 - Optimised 1815 RCBC with trimmed voids and reinforcement

If the impossible voids are removed, the final optimised culvert would look like shown in Figure 5-15.

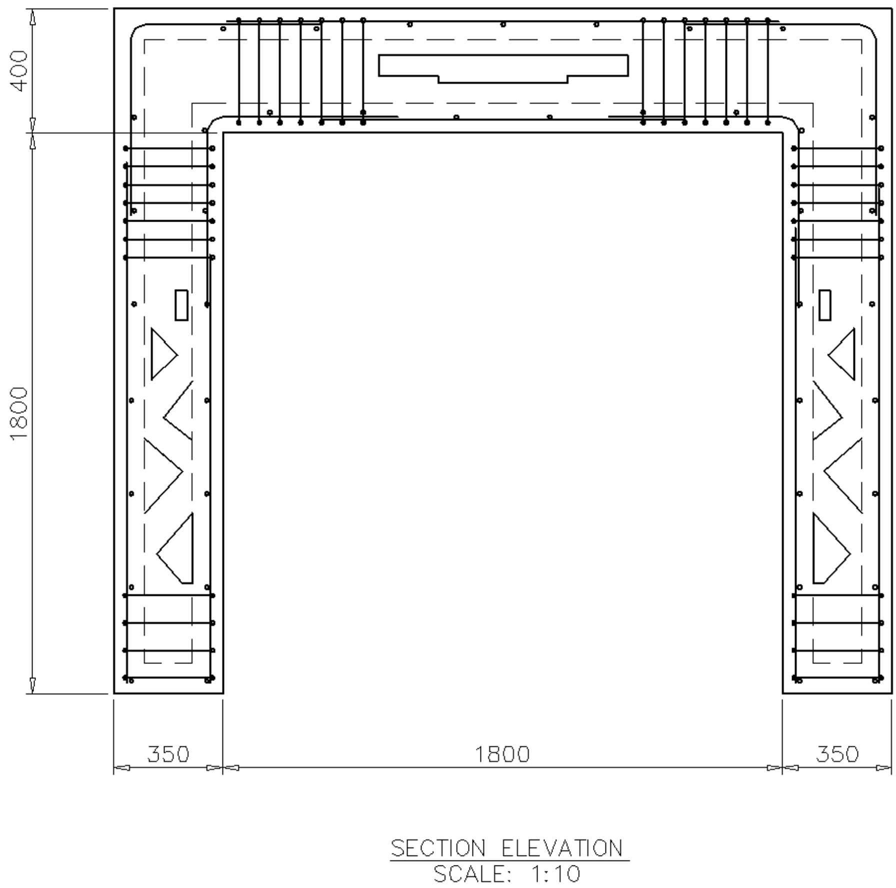

Figure 5-10 - Final 1818 optimised RCBC

5.4 2412 RCBC

The topology optimisation Matlab program is applied to the 2412 RCBC with the following input parameters:

span in mm = 2400

leg height in mm = 1200

volume constraint = 0.25

penalisation factor = 3

filter size divided by the element size (r_{min}) = 1.2

The command `top_rcbc4(2400,1200,0.25,3,1.2)` yields the result shown in Figure 5-11. The simulation comprised of 116 iterations and took 119 seconds.

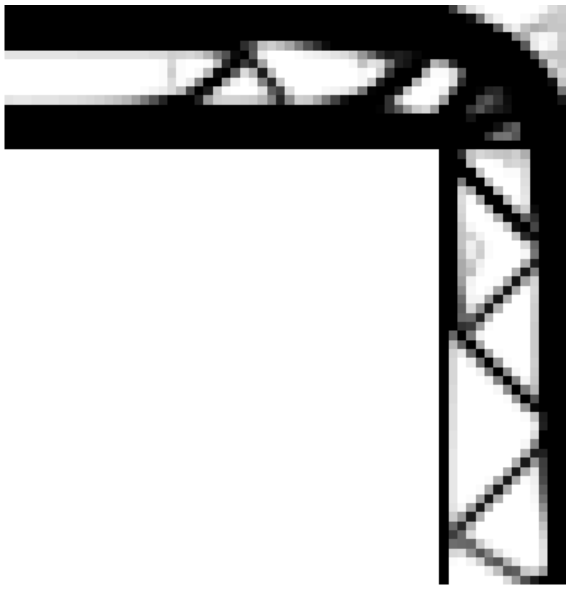

Figure 5-11 - Topology optimisation result for 2412 RCBC

The voids in the optimised culvert are to be created only where the colour map shows white, that is, where the relative density is 0.001. Grey areas mean composite materials and since this study is not going to utilise composite materials, only the white parts can become voids. The culvert would look like Figure 5-12.

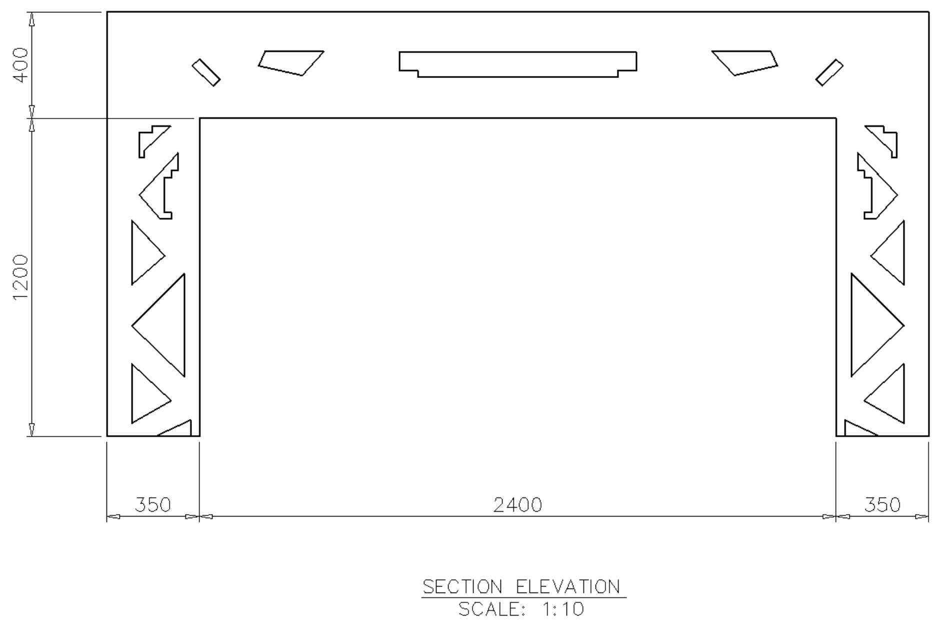

Figure 5-12 – Optimised 2412 RCBC section elevation

However, it can be seen the voids are at times too close to the edge of the culvert. We know the leg reinforcement for the 1815 RCBC leg is comprised of N16 bars with N12 distribution bars. That means the distance between the edge of the void to the edge of the leg needs to be a minimum of 35+35+20+12=102 mm, which is shown by the dashed line in Figure 5-13.

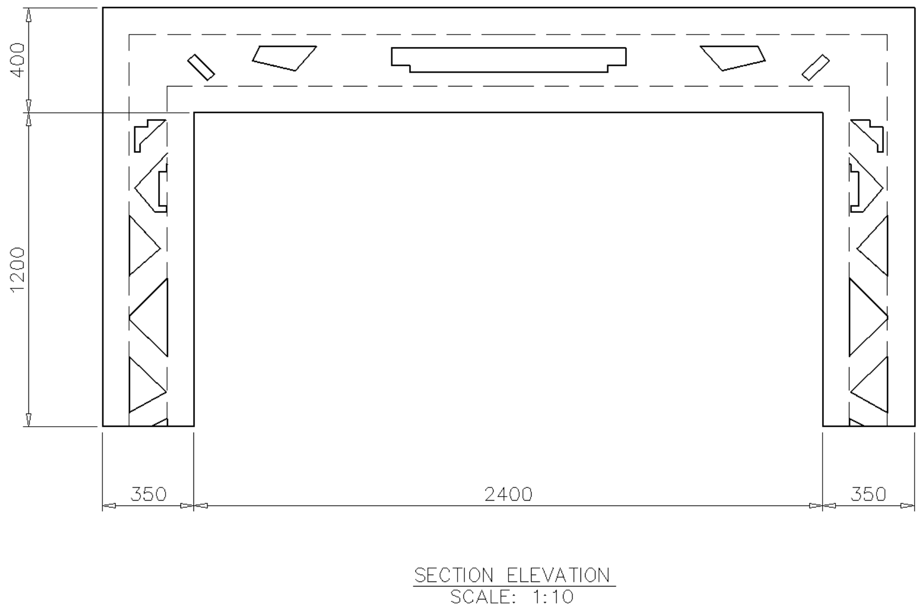

Figure 5-13 - Optimised 2412 RCBC with trimmed voids

The voids then need to be trimmed to be within the dashed line. Also, the reinforcement has to fit in and there needs to be 35 mm cover at all times. The 2412 RCBC with trimmed voids is shown in Figure 5-14 with superimposed reinforcement. It can be seen that some voids will not be possible because they coincide with the shear ligatures.

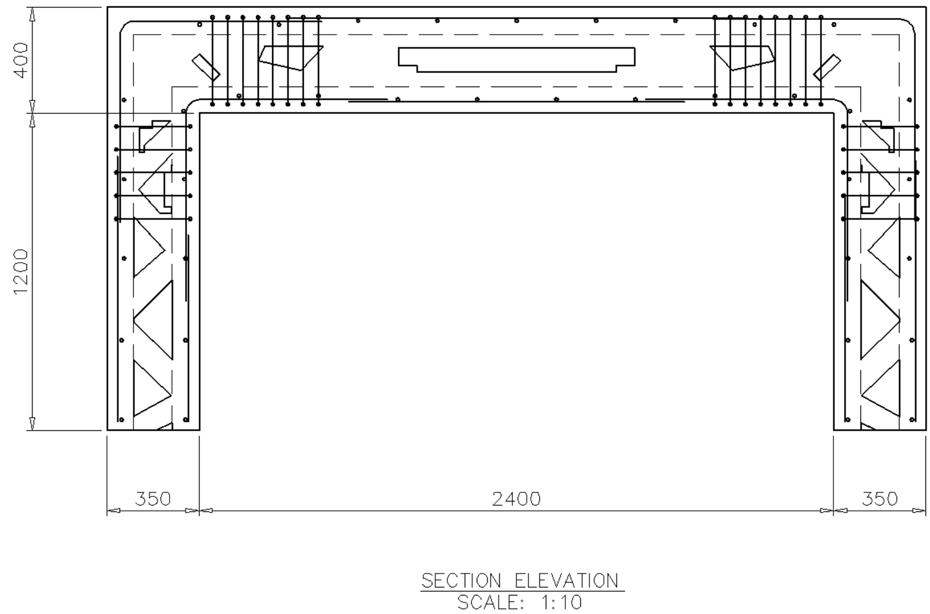

Figure 5-14 - Optimised 2412 RCBC with trimmed voids and reinforcement

If the impossible voids are removed, the final optimised culvert would look like shown in Figure 5-15.

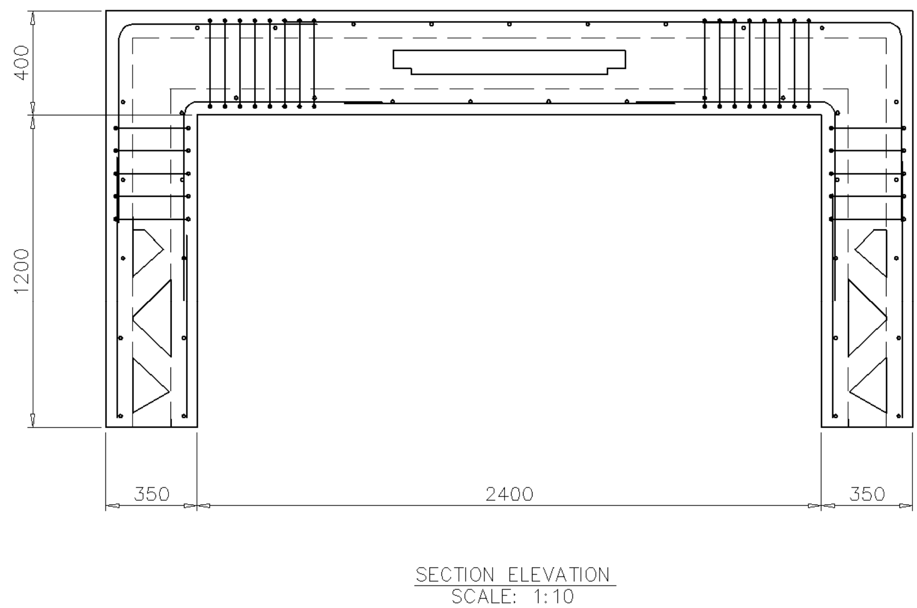

Figure 5-15 - Final 2412 optimised RCBC

5.5 2415 RCBC

The topology optimisation Matlab program is applied to the 2415 RCBC with the following input parameters:

span in mm = 2400
leg height in mm = 1500
volume constraint = 0.25
penalisation factor = 3
filter size divided by the element size (r_{min}) = 1.2

The command `top_rcbc4(2400,1500,0.25,3,1.2)` yields the result shown in Figure 5-16. The simulation comprised of 125 iterations and took 236 seconds.

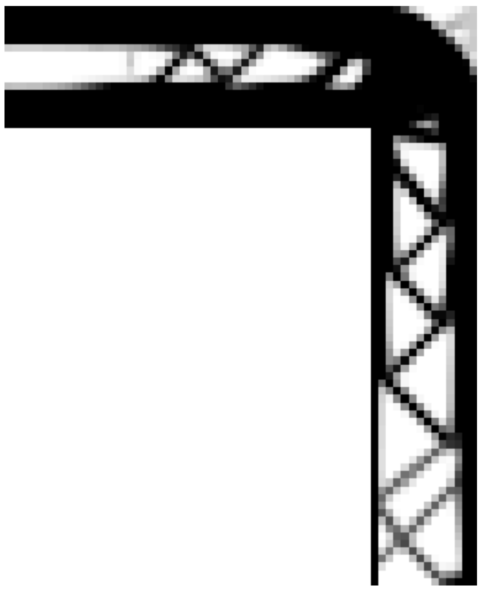

Figure 5-16 - Topology optimisation result for 2415 RCBC

The voids in the optimised culvert are to be created only where the colour map shows white, that is, where the relative density is 0.001. Grey areas mean composite materials and since this study is not going to utilise composite

materials, only the white parts can become voids. The culvert would look like Figure 5-17.

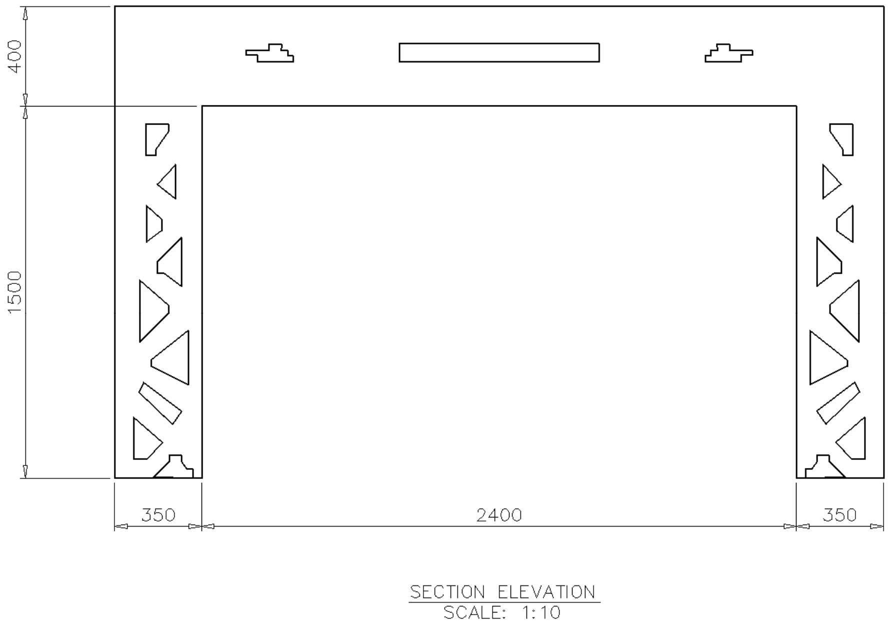

Figure 5-17 – Optimised 2415 RCBC section elevation

However, it can be seen the voids are at times too close to the edge of the culvert. We know the leg reinforcement for the 2415 RCBC leg is comprised of N20 bars with N12 distribution bars. That means the distance between the edge of the void to the edge of the leg needs to be a minimum of 35+35+20+12=102 mm, which is shown by the dashed line in Figure 5-18.

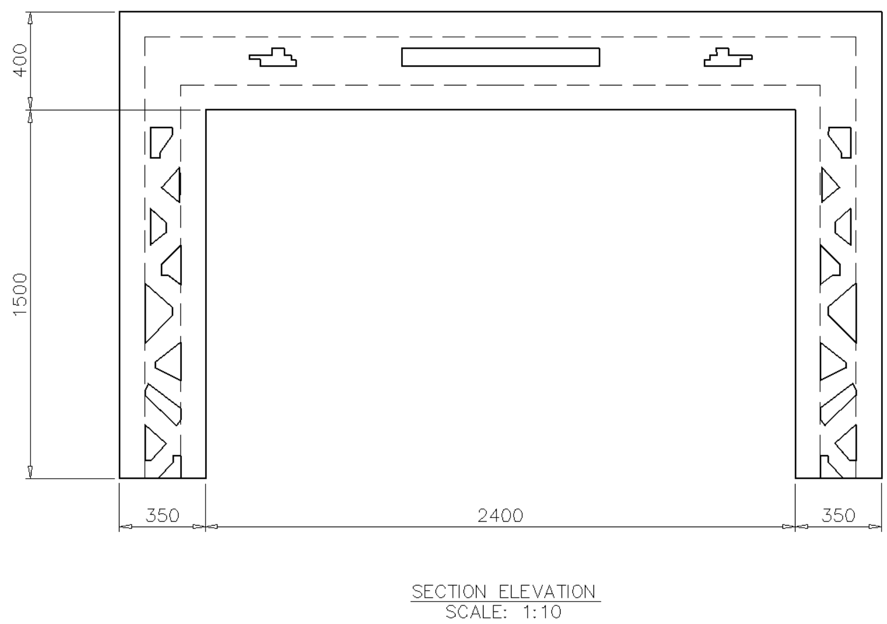

Figure 5-18 - Optimised 2415 RCBC with trimmed voids

The voids then need to be trimmed to be within the dashed line. Also, the reinforcement has to fit in and there needs to be 35 mm cover at all times. The 2415 RCBC with trimmed voids is shown in Figure 5-19 with superimposed reinforcement. It can be seen that some voids will not be possible because they coincide with the shear ligatures.

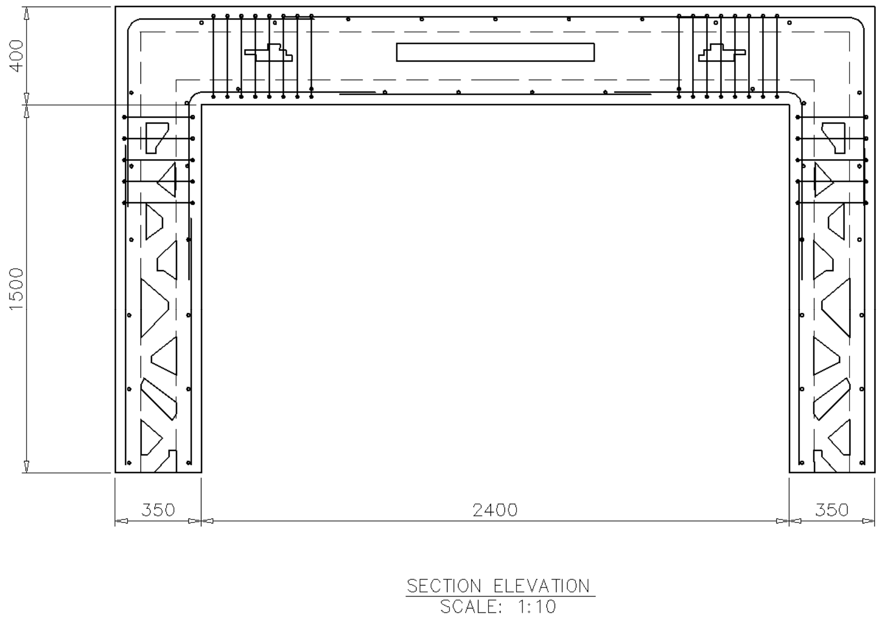

Figure 5-19 - Optimised 2415 RCBC with trimmed voids and reinforcement

If the impossible voids are removed, the final optimised culvert would look like shown in Figure 5-20.

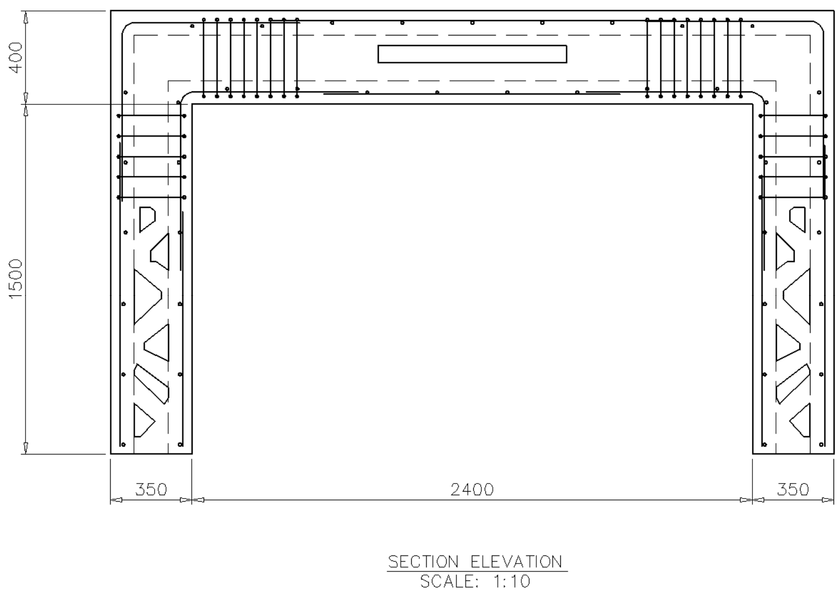

Figure 5-20 - Final 2418 optimised RCBC

5.6 2418 RCBC

The topology optimisation Matlab program is applied to the 2418 RCBC with the following input parameters:

span in mm = 2400
leg height in mm = 1800
volume constraint = 0.25
penalisation factor = 3
filter size divided by the element size (r_{min}) = 1.2

The command `top_rcbc4(2400,1800,0.25,3,1.2)` yields the result shown in Figure 5-21. The simulation comprised of 153 iterations and took 278 seconds.

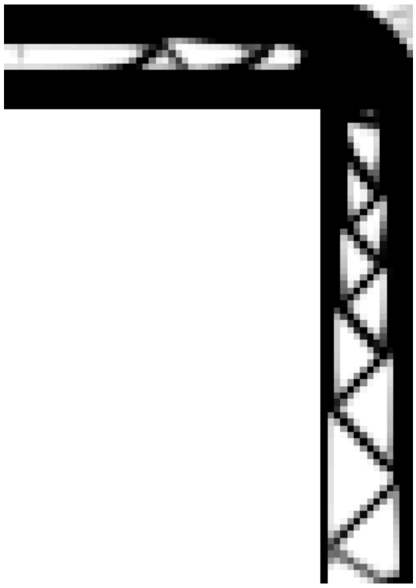

Figure 5-21 - Topology optimisation result for 2418 RCBC

The voids in the optimised culvert are to be created only where the colour map shows white, that is, where the relative density is 0.001. Grey areas mean composite materials and since this study is not going to utilise composite

materials, only the white parts can become voids. The culvert would look like Figure 5-22.

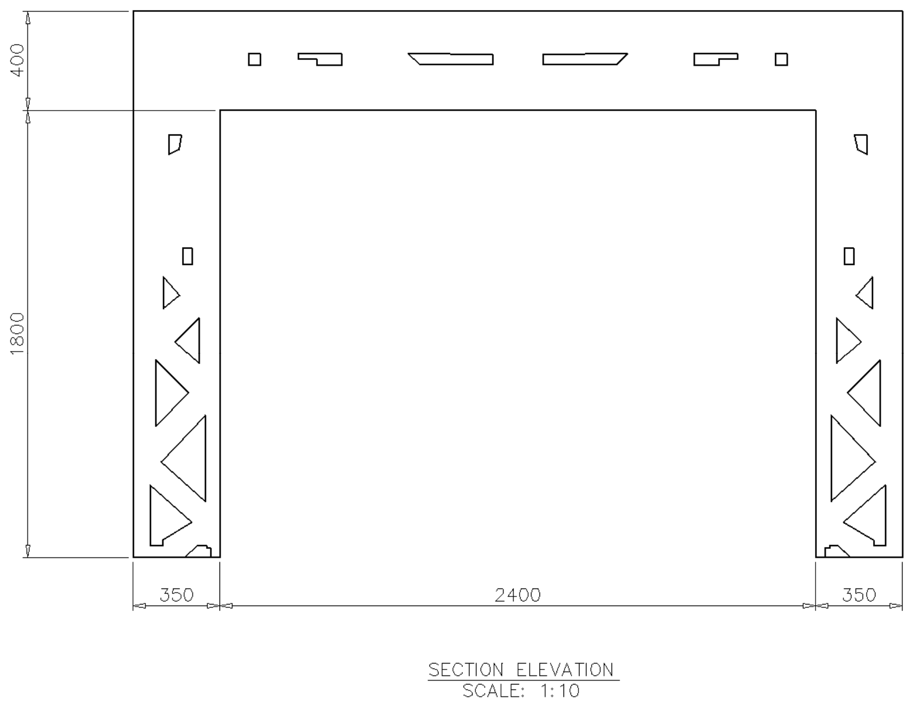

Figure 5-22 – Optimised 2418 RCBC section elevation

However, it can be seen the voids are at times too close to the edge of the culvert. We know the leg reinforcement for the 2418 RCBC leg is comprised of N20 bars with N12 distribution bars. That means the distance between the edge of the void to the edge of the leg needs to be a minimum of 35+35+20+12=102 mm, which is shown by the dashed line in Figure 5-23.

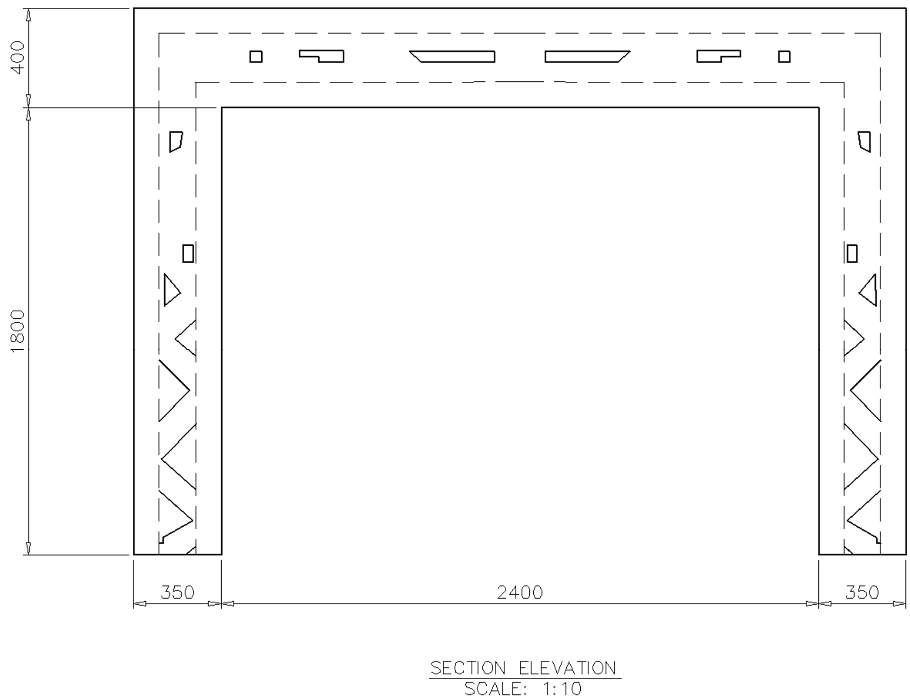

Figure 5-23 - Optimised 2418 with trimmed voids

The voids then need to be trimmed to be within the dashed line. Also, the reinforcement has to fit in and there needs to be 35 mm cover at all times. The 2418 RCBC with trimmed voids is shown in Figure 5-24 with superimposed reinforcement. It can be seen that some voids will not be possible because they coincide with the shear ligatures.

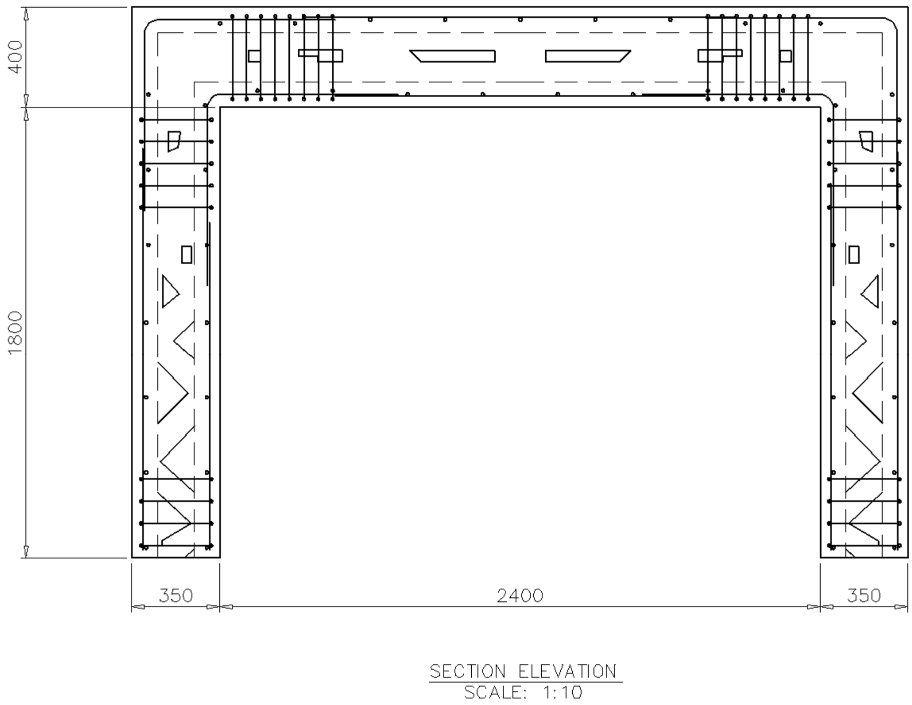

Figure 5-24 - Optimised 2418 RCBC with trimmed voids and reinforcement

If the impossible voids are removed, the final optimised culvert would look like shown in Figure 5-25.

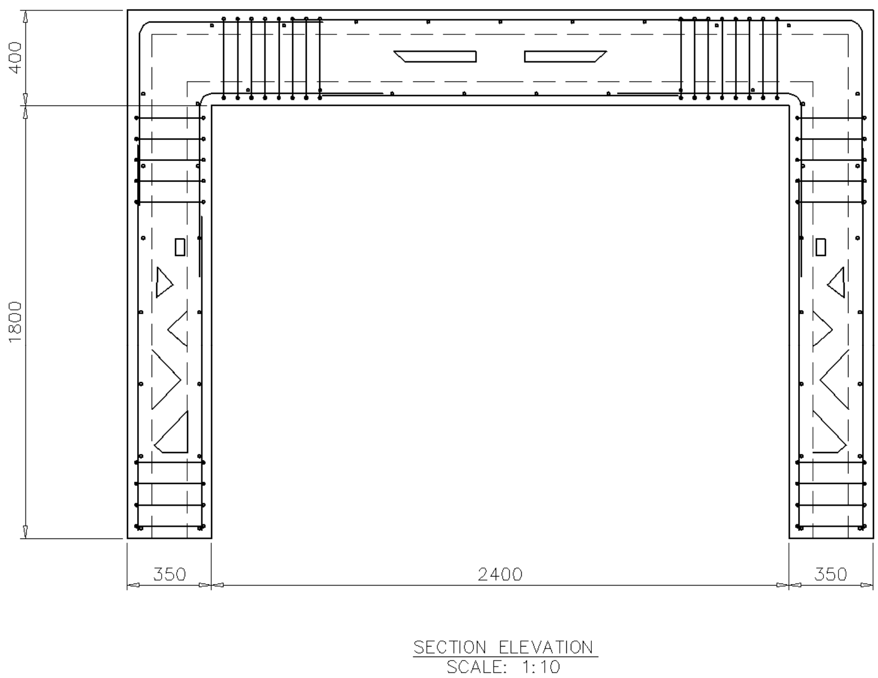

Figure 5-25 - Final 2418 optimised RCBC

5.7 Conclusion

The topology optimisation procedure is greatly simplified by the usage of the script adapted from Sigmund (2001). Great results have been obtained that did not present checkerboards or too much composite material and voids were able to be used in the culverts without issues. Also, the computation time is not significant and is not likely to be prohibitive to the procedure's adoption. The presence of shear ligatures greatly influences the final topology of the structure, in this case. Any decrease in the need for shear ligatures will likely result in great usage of voids in the culverts.

The successful application of this procedure as was done in this study depends on some dealing with Matlab and some drafting in AutoCad, which are two things most engineers should be familiar with. Therefore, it is safe to say most engineers would not have difficulty with the procedure and would get familiar with it quickly and be able to efficiently apply the procedure to a variety of RCBCs.

The adoption of this procedure by manufacturers would greatly depend on feasibility of the whole exercise, which is analysed and discussed in the next section.

CHAPTER 6 - FEASIBILITY ANALYSIS

6.1 Introduction

Because one of the focus points of this project is to draw conclusions regarding the optimum topology of RCBC commonly used in the industry, it is paramount that costs are analysed to indicate the feasibility of the method. During the design phase, the costs and time required to perform the SIMP analysis will be compared with estimated costs and time required to run a similar design for a standard RCBC sold by manufacturers. In relation to manufacture, the complexity and cost of the reinforcement cage required will be analysed and compared with estimated costs used by manufacturers to produce non-optimised culverts. Regarding transportation, the cost and time required to load, unload and transport the optimum RCBC will be compared between the optimised and non-optimised culverts.

These results will be contrasted with the benefits that using this optimisation procedure may bring and a conclusion will be reached regarding feasibility of designing, manufacturing and installing the optimum RCBC.

6.2 Labour and design components of total cost

Because the culverts were not very dissimilar in size, cost changes to allow for labour and design costs for the optimised culvert were equally applied through all culverts. Extra labour hours would be required to design the optimised culverts and to organise the voids. It is important to note that the labour and overhead costs are calculated based on the mass of the entire unit. The reinforcement is exactly the same, since the voids are trimmed to fit the reinforcement configuration. The loading procedures would also be the same since the centre of gravity of the optimised units was only raised by a few millimetres to a maximum of 24 mm and that would not pose any problem to transporting the culverts legs down on the truck. It was assumed in this study that the cost of labour would increase in 20% to allow for these extra required activities. As per discussed in Section 3.5.2, one man-hour is assumed to cost \$40.

In addition, the optimised culvert was assumed to take twice as long to design than the non-optimised culvert because once the non-optimised culvert has been found, the topology optimisation procedure has to be carried out and will include some designing and some drafting. However, this assumption leans on the conservative side because once the designer is used to the software utilised to carry out the optimisation, design time will become shorter and shorter. Also, it is possible to improve the optimisation programmes that were utilised for this study by condensing all functions into one or two programmes to increase ease of use. That would also mean design time could be reduced.

6.3 1815 RCBC

6.3.1 Materials

The non-optimised 1815 RCBC would utilise 4.92 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 508.6 kg and the density of the steel is assumed 7850 kg/m^3 (Standards Australia 2001, p. 14). Therefore, 508.6 kg of steel would displace 0.0648 $m^3$of concrete. The total concrete volume would then be 4.86 m^3, which at a density of 2400 kg/m^3 equates to 11.657 tonnes. The total mass of the culvert is therefore 12.166 tonnes.

The optimised 1815 RCBC would utilise 4.55 m^3 of concrete minus the 0.0628 m^3 displaced by the reinforcement, which gives 4.48 m^3. At a density of 2400 kg/m^3, this equates to 10.758 tonnes. The total mass of the culvert is therefore 11.251 tonnes. The difference is 899 kg of concrete, representing 7.4% of the total initial amount of concrete, before the optimisation.

6.3.2 Conclusion

The pertinent parameters namely materials, design, labour and overheads for the non-optimised and optimised RCBCs were compiled into Table 6-1 and Table 6-2.

Table 6-1 - Cost summary of non-optimised 1815 RCBC

COSTS OF NON-OPTIMISED 1815 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	1.5	hours	80	$/hour of design	$ 120.00
Materials: Steel	508.6	kg	1300	$/tonne of steel	$ 661.18
Materials: Concrete	4.86	m3	140	$/m3 of concrete	$ 680.40
Labour			2.5	man-hours/tonne	$ 1,217.26
Overheads			70	$/tonne	$ 852.08
				TOTAL COST:	$ 3,530.92

Table 6-2 - Cost summary of optimised 1815 RCBC

COSTS OF OPTIMISED 1815 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	3	hours	80	$/hour of design	$ 240.00
Materials: Steel	508.6	kg	1300	$/tonne of steel	$ 661.18
Materials: Concrete	4.48	m3	140	$/m3 of concrete	$ 627.20
Labour			3	man-hours/tonne	$ 1,351.27
Overheads			70	$/tonne	$ 788.24
				TOTAL COST:	$ 3,667.89

The total manufacture cost of the optimised culvert is approximately 3.9% higher than the non-optimised culvert.

6.4 1818 RCBC

6.4.1 Materials

Regarding materials, the non-optimised 1818 RCBC would utilise 5.43 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 663.33 kg and the density of the steel is assumed 7850 kg/m^3 (Standards Australia 2001, p. 14). Therefore, 663.33 kg of steel would displace 0.0845 $m^3$of concrete. The total concrete volume would then be 5.35 m^3, which at a density of 2400 kg/m^3 equates to 12.829 tonnes. The total mass of the culvert is therefore 13.493 tonnes.

The optimised 1818 RCBC would utilise 5.05 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 663.33 kg and it would displace 0.0845 $m^3$of concrete. The total concrete volume would then be 4.97 m^3, which at a density of 2400 kg/m^3 equates to 11.917 tonnes. The total mass of the culvert is therefore 12.581 tonnes. The difference is 912 kg of concrete, representing 6.8% of the total initial amount of concrete, before the optimisation.

Extra labour hours would be required to design the optimised culvert and to organise the voids. The reinforcement is be exactly the same, since the voids were trimmed to fit the reinforcement configuration. The loading procedures would also be the same since the centre of gravity of the optimised unit was only raised by 10 mm towards the crown at 1.386 m from the leg, and that would not pose any problem to transporting it legs down on the truck.

6.4.2 Conclusion

The pertinent parameters namely materials, design, labour and overheads for the non-optimised and optimised RCBCs were compiled into Table 6-3 and Table 6-4.

Table 6-3 - Cost summary of non-optimised 1818 RCBC

COSTS OF NON-OPTIMISED 1818 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	1.5	hours	80	$/hour of design	$ 120.00
Materials: Steel	663.3	kg	1300	$/tonne of steel	$ 862.33
Materials: Concrete	5.35	m3	140	$/m3 of concrete	$ 749.00
Labour			2.5	man-hours/tonne	$ 1,350.33
Overheads			70	$/tonne	$ 945.23
				TOTAL COST:	$ 4,026.90

Table 6-4 - Cost summary of optimised 1818 RCBC

COSTS OF OPTIMISED 1818 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	3	hours	80	$/hour of design	$ 240.00
Materials: Steel	663.3	kg	1300	$/tonne of steel	$ 862.33
Materials: Concrete	4.97	m3	140	$/m3 of concrete	$ 695.80
Labour			3	man-hours/tonne	$ 1,510.96
Overheads			70	$/tonne	$ 881.39
				TOTAL COST:	$ 4,190.48

The total manufacture cost of the optimised culvert is approximately 4.06% higher than the non-optimised culvert.

6.5 2412 RCBC

6.5.1 Materials

The non-optimised 2412 RCBC would utilise 4.99 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 725.49 kg and the density of the steel is assumed 7850 kg/m^3 (Standards Australia 2001, p. 14). Therefore, 725.49 kg of steel would displace 0.0924 $m^3$of concrete. The total concrete volume would then be 4.90 m^3, which at a density of 2400 kg/m^3 equates to 11.754 tonnes. The total mass of the culvert is therefore 12.480 tonnes.

The optimised 2412 RCBC would utilise 4.56 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 725.49 kg and it would displace 0.0924 $m^3$of concrete. The total concrete volume would then be 4.47 m^3, which at a density of 2400 kg/m^3 equates to 10.722 tonnes. The total mass of the culvert is therefore 11.448 tonnes. The difference is 1032 kg of concrete, representing 8.8% of the total initial amount of concrete, before the optimisation.

6.5.2 Conclusion

The pertinent parameters namely materials, design, labour and overheads for the non-optimised and optimised RCBCs were compiled into Table 6-5 and Table 6-6.

Table 6-5 - Cost summary of non-optimised 2412 RCBC

COSTS OF NON-OPTIMISED 2412 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	1.5	hours	80	$/hour of design	$ 120.00
Materials: Steel	725.5	kg	1300	$/tonne of steel	$ 943.14
Materials: Concrete	4.9	m3	140	$/m3 of concrete	$ 686.00
Labour			2.5	man-hours/tonne	$ 1,248.55
Overheads			70	$/tonne	$ 873.98
				TOTAL COST:	$ 3,871.67

Table 6-6 - Cost summary of optimised 2412 RCBC

COSTS OF OPTIMISED 2412 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	3	hours	80	$/hour of design	$ 240.00
Materials: Steel	725.5	kg	1300	$/tonne of steel	$ 943.14
Materials: Concrete	4.47	m3	140	$/m3 of concrete	$ 625.80
Labour			3	man-hours/tonne	$ 1,374.42
Overheads			70	$/tonne	$ 801.74
				TOTAL COST:	$ 3,985.10

The total manufacture cost of the optimised culvert is approximately 2.95% higher than the non-optimised culvert.

6.6 2415 RCBC

6.6.1 Materials

The non-optimised 2415 RCBC would utilise 5.50 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 725.24 kg and the density of the steel is assumed 7850 kg/m^3 (Standards Australia 2001, p. 14). Therefore, 725.24 kg of steel would displace 0.0924 $m^3$of concrete. The total concrete volume would then be 5.41 m^3, which at a density of 2400 kg/m^3 equates to 12.978 tonnes. The total mass of the culvert is therefore 13.704 tonnes.

The optimised 2415 RCBC would utilise 5.09 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 725.24 kg and it would displace 0.0924 $m^3$of concrete. The total concrete volume would then be 5.00 m^3, which at a density of 2400 kg/m^3 equates to 11.994 tonnes. The total mass of the culvert is therefore 12.719 tonnes. The difference is 984 kg of concrete, representing 7.6% of the total initial amount of concrete, before the optimisation.

6.6.2 Conclusion

The pertinent parameters namely materials, design, labour and overheads for the non-optimised and optimised RCBCs were compiled into Table 6-7 and Table 6-8.

Table 6-7 - Cost summary of non-optimised 2415 RCBC

COSTS OF NON-OPTIMISED 2415 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	1.5	hours	80	$/hour of design	$ 120.00
Materials: Steel	725.2	kg	1300	$/tonne of steel	$ 942.81
Materials: Concrete	5.41	m3	140	$/m3 of concrete	$ 757.40
Labour			2.5	man-hours/tonne	$ 1,370.92
Overheads			70	$/tonne	$ 959.65
				TOTAL COST:	$ 4,150.78

Table 6-8 - Cost summary of optimised 2415 RCBC

COSTS OF OPTIMISED 2415 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	3	hours	80	$/hour of design	$ 240.00
Materials: Steel	725.2	kg	1300	$/tonne of steel	$ 942.81
Materials: Concrete	5	m3	140	$/m3 of concrete	$ 700.00
Labour			3	man-hours/tonne	$ 1,527.03
Overheads			70	$/tonne	$ 890.77
				TOTAL COST:	$ 4,300.61

The total manufacture cost of the optimised culvert is approximately 3.60% higher than the non-optimised culvert.

6.7 2418 RCBC

6.7.1 Materials

The non-optimised 2418 RCBC would utilise 6.0 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 840.13 kg and the density of the steel is assumed 7850 kg/m^3 (Standards Australia 2001, p. 14). Therefore, 840.13 kg of steel would displace 0.107 $m^3$of concrete. The total concrete volume would then be 5.89 m^3, which at a density of 2400 kg/m^3 equates to 14.143 tonnes. The total mass of the culvert is therefore 14.983 tonnes.

The optimised 2418 RCBC would utilise 5.74 m^3 of concrete minus what would be displaced by the reinforcement. The reinforcement was found to weigh 840.13 kg and it would displace 0.107 $m^3$of concrete. The total concrete volume would then be 5.634 m^3, which at a density of 2400 kg/m^3 equates to 13.522 tonnes. The total mass of the culvert is therefore 14.362 tonnes. The difference is 621 kg of concrete, representing 4.4% of the total initial amount of concrete, before the optimisation.

6.7.2 Conclusion

The pertinent parameters namely materials, design, labour and overheads for the non-optimised and optimised RCBCs were compiled into Table 6-9 and Table 6-10.

Table 6-9 - Cost summary of non-optimised 2418 RCBC

COSTS OF NON-OPTIMISED 2418 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	1.5	hours	80	$/hour of design	$ 120.00
Materials: Steel	840.1	kg	1300	$/tonne of steel	$ 1,092.17
Materials: Concrete	5.89	m3	140	$/m3 of concrete	$ 824.60
Labour			2.5	man-hours/tonne	$ 1,497.61
Overheads			70	$/tonne	$ 1,048.33
				TOTAL COST:	$ 4,582.71

Table 6-10 - Cost summary of optimised 2418 RCBC

COSTS OF OPTIMISED 2418 RCBC					
PARAMATER	QTY	UNIT	COST PER UNIT		TOTAL COST ($)
Design	3	hours	80	$/hour of design	$ 240.00
Materials: Steel	840.1	kg	1300	$/tonne of steel	$ 1,092.17
Materials: Concrete	5.634	m3	140	$/m3 of concrete	$ 788.76
Labour			3	man-hours/tonne	$ 1,723.41
Overheads			70	$/tonne	$ 1,005.32
				TOTAL COST:	$ 4,849.66

The total manufacture cost of the optimised culvert is approximately 5.85% higher than the non-optimised culvert.

6.8 Conclusions

The difference between the cost of the non-optimised culvert and the optimised culvert seems closely related to the leg height. The higher the leg height the higher the difference. Similar leg heights presented similar differences: for the 1815 the difference was 3.9% and for the 2415 the difference was 3.6% while for the 1818 the difference was 4.06% and for the 2418 it was 8.85%. This would need to be confirmed by performing an analysis of all the 24 sizes of culverts, but it would make sense that the price is related to the leg height since it is in the leg that the majority of the voids is located.

Labour is the biggest factor in the cost of an RCBC, representing from 32% to 37% of the costs of the culvert over the 5 units studied. As pointed out by Roome (2014), labour would be a key parameter to reduce the costs of the RCBC.

Design represents between 2.9% and 3.4% of the costs of the non-optimised unit and between 5.0% and 6.6% of the costs of the optimised unit. This means that efforts put towards diminishing design time of the optimised culvert will yield much smaller savings than those put towards standardization of the optimised units to save on labour costs.

Even though the optimised culvert ended up slightly more expensive than the non-optimised culvert, it is still very possible that it may be feasible for the manufacturers to offer that option to the customers. One reason is to accommodate customers' requirements. If they need slots to pass cables or for some other application these requirements can be met by the optimised culvert. Another reason would be that it can be sold as a more environmentally sustainable option because it saves around 5% in concrete utilisation. Also, if methods to standardise the procedure or to reduce labour costs somehow can be found, it will not take much for the costs of the two culverts to equalise.

CHAPTER 7 - CONCLUSIONS

7.1 Achievements

This project involved the analysis of SM1600 loads over large reinforced concrete box culverts (RCBC); the design of non-optimised RCBC units based on current AS3600-2009 specifications; the determination of the optimum topology for the five most economically significant RCBCs; and a feasibility analysis to determine in which situations the application of the process outlined by this project would be worthwhile.

To aid the accomplishment of these steps, various Matlab programs were written to calculate load combinations, perform flexibility and shear analysis and find critical horizontal and vertical loads acting on the culverts. In addition, a Matlab program was adapted from Sigmund (2001) to perform the topology optimisation utilising the Solid Isotropic Material with Penalization (SIMP) method with finite element analysis.

In order to ensure the results achieved by this study were as applicable as possible to the industry, an interview was carried out with an experienced manager with over 20 years' experience in the precast concrete industry. The information obtained through this interview was invaluable to guaranteeing the models in this study were as close as possible to the real situations in the industry.

7.2 Conclusions

The topology optimisation procedure presented in this project yields good results that can be applied in the industry. It is not a complicated procedure when the designer makes use of programming in the form of Matlab scripts and the like to perform the calculations. Even though this project analysed Queensland Main Roads culverts, the same procedure can be used in other applications such as subdivision culverts, which are subjected to smaller loads and may enable greater inclusion of voids in their topology.

The Solid Isotropic Material with Penalization (SIMP) method yielded clean results, with not many grey areas (composite materials) and without checkerboard issues due to the utilisation of a mesh-independency filter developed by Sigmund (2001). That made it possible to establish where the voids would need to be placed and what size and shape they should be.

The optimised culverts offered a reduction of between 4.4% and 8.8% in the amount of concrete utilised. The steel reinforcement remained the same since reinforcement optimisation is out of the scope of this project. However, due to labour costs, the optimised culverts were estimated to be between 2.95% to 5.85% more expensive to manufacture than the non-optimised culverts. If procedures are put in place to standardise the inclusion of voids in the culverts and thus reduce labour costs, it is very likely that the optimised culverts could cost the same or less to manufacture.

7.3 Possible further work

When examining possible further work, one of the first things that has to be mentioned is the possibility to analyse all culvert sizes to ascertain the level of savings and feasibility outside the studied interval of RCBCs. Despite the fact that they do not represent the majority of sales, it is possible that the exercise is worthwhile depending on the size and scale of the construction job they will be used in.

Another significant project would be to improve and condense the Matlab scripts to increase user-friendliness. A savvy Matlab user would not have much difficulty understanding how each script interacts with the others and how to apply them to the topology optimisation process, but for the designers who only have basic knowledge of Matlab the way the scripts are organised can prove to be challenging to utilise. Any improvement in that area will decrease design time and increase the chance of adoption of the procedure by the industry.

In addition, materials other than concrete and steel can be investigated and different topologies may be found. The usage of fibre reinforced concrete (FRC), for instance, can prove to minimise issues with cover to reinforcement.

In a more practical way, the optimised culverts found by this project could be manufactured and load tested, to confirm assumptions made in this study and provide the industry with proof of its validity.

REFERENCES

Ahmed, AU & Amanat, KM 2008, 'Need of 3d finite element analysis for buried RC box culvert in fill soil', in Annual Conference of the Canadian Society for Civil Engineering 2008 - "Partnership for Innovation", June 10, 2008 - June 13, 2008: *proceedings of theAnnual Conference of the Canadian Society for Civil Engineering 2008 - "Partnership for Innovation", June 10, 2008 - June 13, 2008* Canadian Society for Civil Engineering, Quebec City, QC, Canada, pp. 2078-87.

Aschheim, M, Hernandez-Montes, E & Gil-Martin, LM 2008, 'Design of optimally reinforced RC beam, column, and wall sections', *Journal of Structural Engineering*, vol. 134, no. 2, pp. 231-9.

Awwad, E, Mabsout, M, Sadek, S & Tarhini, K 2000, 'Finite element analysis of concrete box culverts', in 8th International Conference on Computing in Civil and Building Engineering, August 14, 2000 - August 16, 2000: *proceedings of the8th International Conference on Computing in Civil and Building Engineering, August 14, 2000 - August 16, 2000* American Society of Civil Engineers, Stanford, CA, United states, pp. 1051-3, <http://dx.doi.org/10.1061/40513(279)136>.

Awwad, E, Mabsout, M, Sadek, S & Tarhini, K 2008, 'Parametric study of load distribution in four-sided concrete box culverts', *Bridge Structures*, vol. 4, no. 2, pp. 99-107.

Bendsøe, MP & Sigmund, O 1999, 'Material interpolation schemes in topology optimization', *Archive of Applied Mechanics*, vol. 69, no. 9-10, pp. 635-54.

Bruns, TE 2005, 'A reevaluation of the SIMP method with filtering and an alternative formulation for solid-void topology optimization', *Structural and Multidisciplinary Optimization*, vol. 30, no. 6, pp. 428-36.

Chen, B-G, Zheng, J-J & Han, J 2010, 'Experimental study and numerical simulation on concrete box culverts in trenches', *Journal of Performance of Constructed Facilities*, vol. 24, no. 3, pp. 223-34.

China Ministry of Transport 2004, *JTG D60-2004 - General Code for Design of Highway Bridges and Culverts (English)*, China Communications Press, Beijing.

Das, S 2013, 'Application of diagnostic load testing and 3D-FEA in load rating of RC box culverts', *Bridge Structures*, vol. 9, no. 4, pp. 155-67.

Foley Products 2014, *Box Culverts*, <http://www.theconcretecompany.com/test/images/box_culvert-l.jpg>.

Foster, S 2010, *Reinforced concrete basics 2E : analysis and design of reinforced concrete structures*, 2nd edition. edn, Pearson Education Australia, Frenchs Forest NSW.

Frederick, GR & Tarhini, KM 2000, 'Structural evaluation of three-sided concrete culverts', in Concrete Pipe for the New Millennium, May 19, 1999 - May 20, 1999: *proceedings of theConcrete Pipe for the New Millennium, May 19, 1999 - May 20, 1999* ASTM, Seattle, WA, USA, pp. 36-44.

Garg, AK 2006, 'Experimental and finite element based investigations of shear behavior in reinforced concrete box culverts', ProQuest, UMI Dissertations Publishing.

Garg, AK & Abolmaali, A 2009, 'Finite-element modeling and analysis of reinforced concrete box culverts', *Journal of Transportation Engineering*, vol. 135, no. 3, pp. 121-8.

Gil-Martin, LM, Aschheim, M, Hernandez-Montes, E & Pasadas-Fernandez, M 2011, 'Recent developments in optimal reinforcement of RC beam and column sections', *Engineering Structures*, vol. 33, no. 4, pp. 1170-80.

Guest, J & Moen, C 2010, 'Reinforced Concrete Design with Topology Optimization', in *Structures Congress 2010*, pp. 445-54.

Guest, J, Lotfi, R, Gaynor, A & Jalalpour, M 2012, 'Structural Topology Optimization: Moving Beyond Linear Elastic Design Objectives', in *20th Analysis and Computation Specialty Conference*, pp. 245-56.

Guest, JK 2009, 'Topology optimization with multiple phase projection', *Computer Methods in Applied Mechanics and Engineering*, vol. 199, no. 1–4, pp. 123-35.

Guest, JK, Prévost, JH & Belytschko, T 2004, 'Achieving minimum length scale in topology optimization using nodal design variables and projection functions', *International Journal for Numerical Methods in Engineering*, vol. 61, no. 2, pp. 238-54.

Hassani, B & Hinton, E 1998, 'A review of homogenization and topology optimization III—topology optimization using optimality criteria', *Computers & Structures*, vol. 69, no. 6, pp. 739-56.

Huang, X 2010, *Evolutionary topology optimization of continuum structures methods and applications*, Wiley, Chichester West Sussex UK, <http://ezproxy.usq.edu.au/login?url=http://site.ebrary.com/lib/unisouthernqld/Doc?id=10375595>.

Huang, X & Xie, YM 2007, 'Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method', *Finite Elements in Analysis and Design*, vol. 43, no. 14, pp. 1039-49.

Kang, J, Parker, F, Kang, YJ & Yoo, CH 2008, 'Effects of frictional forces acting on sidewalls of buried box culverts', *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 32, no. 3, pp. 289-306.

Kim, D, Bhowmik, SK, Willmer, JL & Leo, E 2011, 'A case history and finite element modeling of a culvert failure', in Geo-Frontiers 2011: Advances in Geotechnical Engineering, March 13, 2011 - March 16, 2011: *proceedings of theGeo-Frontiers 2011: Advances in Geotechnical Engineering, March 13, 2011 - March 16, 2011* American Society of Civil Engineers (ASCE), Dallas, TX, United states, pp. 1641-50, <http://dx.doi.org/10.1061/41165(397)168>.

Kitane, Y & McGrath, TJ 2006, 'Three-dimensional modeling of live loads on culverts', in Pipelines 2006, July 30, 2006 - August 2, 2006: *proceedings of thePipelines 2006, July 30, 2006 - August 2, 2006* American Society of Civil Engineers, Chicago, IL, United states, p. 82, <http://dx.doi.org/10.1061/40854(211)82>.

Loo, Y-C 2010, *Reinforced and prestressed concrete analysis and design with emphasis on application of AS3600-2009*, Cambridge University Press, Leiden, <http://ezproxy.usq.edu.au/login?url=http://www.USQ.eblib.com.au/EBLWeb/patron?target=patron&extendedid=P_581067_0>.

Martín, I, Adeli, H & Sarma, K 1999, 'Cost Optimization of Concrete Structures', *Journal of Structural Engineering*, vol. 125, no. 5, pp. 574-5.

McGrath, TJ, Liepins, AA & Beaver, JL 2005, 'Live load distribution widths for reinforced concrete box sections', in Transportation Research Board - 6th International Bridge Engineering Conference: Reliability, Security, and Sustainability in Bridge Engineering, July 17, 2005 - July 20, 2005: *proceedings of theTransportation Research Board - 6th International Bridge Engineering Conference: Reliability, Security, and Sustainability in Bridge Engineering, July 17, 2005 - July 20, 2005* Transportation Research Board, Boston, MA, United states, pp. 99-108.

Rath, DP, Ahlawat, AS & Ramaswamy, A 1999, 'Shape optimization of RC flexural members', *Journal of structural engineering New York, N.Y.*, vol. 125, no. 12, pp. 1439-46.

Rocla 2014, 'Rocla Box Culvert', viewed 09/09/2014.

Roome, C 2014, *RCBC Production Costs* 14/10/14.

Rozvany, GIN 2001, 'Aims, scope, methods, history and unified terminology of computer-aided topology optimization in structural mechanics', *Structural and Multidisciplinary Optimization*, vol. 21, no. 2, pp. 90-108.

Sarma, KC & Adeli, H 1998, 'Cost optimization of concrete structures', *Journal of structural engineering New York, N.Y.*, vol. 124, no. 5, pp. 570-8.

Shojaee, S & Mohammadian, M 2012, 'Structural topology optimization using an enhanced level set method', *Scientia Iranica*, vol. 19, no. 5, pp. 1157-67.

Sigmund, O 2001, 'A 99 line topology optimization code written in Matlab', *Structural and Multidisciplinary Optimization*, vol. 21, no. 2, pp. 120-7.

Sigmund, O & Petersson, J 1998, 'Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboards, mesh-dependencies and local minima', *Structural optimization*, vol. 16, no. 1, pp. 68-75.

Sigmund, O & Maute, K 2013, 'Topology optimization approaches', *Structural and Multidisciplinary Optimization*, vol. 48, no. 6, pp. 1031-55.

Standards Australia 2001, *AS4671-2001 - Steel Reinforcing Materials*, SAI Global Limites, Sydney, NSW.

Standards Australia 2004a, *AS5100.2-2004 - Bridge Design*, Part 2: Design Loads, SAI Global Limited, Sydney, NSW.

Standards Australia 2004b, *AS5100.5 - Beidge Design Part 5: Concrete*, SAI Global Limited, Sydney, NSW.

Standards Australia 2007, *AS 5100.2 Supp 1-2007 : Bridge design - Design loads - Commentary (Supplement to AS 5100.2 - 2004).*

Standards Australia 2009, *AS3600-2009 - Concrete structures*, SAI Global Limites, Sydney, NSW.

Standards Australia 2013, *AS1597.2-2013 - Precast reinforced concrete box culverts*, Part 2: Large culverts (exceeding 1200 mm span or 1200 mm height and up to and including 4200 mm span and 4200 mm height), SAI Global Limited, Sydney, NSW.

Stanton, A & Javadi, AA 2014, 'An automated approach for an optimised least cost solution of reinforced concrete reservoirs using site parameters', *Engineering Structures*, vol. 60, pp. 32-40.

Strand7 Pty. Ltd. 2010, *Using Strand7 - Introduction to the Strand7 Finite Element Analysis System*, Sydney.

Stromberg, L, Beghini, A, Baker, W & Paulino, G 2011, 'Application of layout and topology optimization using pattern gradation for the conceptual design of buildings', *Structural and Multidisciplinary Optimization*, vol. 43, no. 2, pp. 165-80.

Transport and Main Roads 2010a, *Technical Note 20a - Design Criteria for Large Box Culverts to MRTS24*.

Transport and Main Roads, *Design Criteria for Large Box Culverts to MRTS24,* 2010b.

Van Dijk, NP, Maute, K, Langelaar, M & Van Keulen, F 2013, 'Level-set methods for structural topology optimization: A review', *Structural and Multidisciplinary Optimization*, vol. 48, no. 3, pp. 437-72.

Wang, X, Wang, MY & Guo, D 2004, 'Structural shape and topology optimization in a level-set-based framework of region representation', *Structural and Multidisciplinary Optimization*, vol. 27, no. 1-2, pp. 1-19.

Yamada, T, Izui, K, Nishiwaki, S & Takezawa, A 2010, 'A topology optimization method based on the level set method incorporating a fictitious interface energy', *Computer Methods in Applied Mechanics and Engineering*, vol. 199, no. 45–48, pp. 2876-91.

Zhao, H, Long, K & Ma, ZD 2010, 'Homogenization Topology Optimization Method Based on Continuous Field', *Computers and Structures*, vol. 2010, no. 6, pp. 1-7.

Zhu, M, Yang, Y, Gaynor, A & Guest, J 2014, 'Considering Constructability in Structural Topology Optimization', in *Structures Congress 2014*, pp. 2754-64.

Zhu, P, Liu, R, Liu, W & Wu, X 2012, 'Study on optimal design of a box-culvert under road', in 2012 International Conference on Civil Engineering and Urban Planning, CEUP, August 18, 2012 - August 20, 2012: *proceedings of the2012 International Conference on Civil Engineering and Urban Planning, CEUP, August 18, 2012 - August 20, 2012* American Society of Civil Engineers (ASCE), Yantai, China, pp. 614-8, <http://dx.doi.org/10.1061/9780784412435.110>.

APPENDICES

APPENDIX A – PROJECT SPECIFICATION

The latest version of the project specification is as below.

University of Southern Queensland
FACULTY OF ENGINEERING AND SURVEYING
ENG4111/4112 Research Project
PROJECT SPECIFICATION

FOR:	JULIANA SWEENEY
TOPIC:	Topology optimisation of large reinforced concrete box culverts under SM1600 loads
SUPERVISOR:	Dr Sourish Banerjee (USQ)
PROJECT AIM:	This project aims to find the optimum topology of a large reinforced concrete box culvert (RCBC) under SM1600 loads using finite element analysis and Solid Isotropic Material with Penalization (SIMP) method. A feasibility analysis will then be conducted to ascertain how much can be saved using the optimum design and how practical it would be to design, manufacture and install it in practice.
PROGRAMME:	Issue F, 16/10/14

1. Read and analyse literature relating to optimisation of reinforced concrete products to produce a literature review.
2. Generate 3D finite-element models of standard reinforced concrete large box culvert (RCBC) sizes and design them as per AS1597.2-2013 under SM1600 loads.
3. Use the Solid Isotropic Material with Penalization (SIMP) method to optimise the RCBC topology
4. Perform a feasibility analysis of the optimum box culvert to ascertain the level of efficiency and cost savings in designing, manufacturing and installing the optimised structure.

As time permits:

5. Re-analyse culverts including shear reinforcement and draw conclusions regarding its influence in the optimum design

AGREED:

Student: Juliana Sweeney __________ Date: 16/10/2014

Supervisor: Dr. Sourish Banerjee __________ Date: 27/10/2014

A typed version is included on the next page, to facilitate reading.

University of Southern Queensland
FACULTY OF ENGINEERING AND SURVEYING
ENG4111/4112 Research Project
PROJECT SPECIFICATION

FOR:	JULIANA SWEENEY
TOPIC:	Topology optimisation of large reinforced concrete box culverts under SM1600 loads
SUPERVISOR:	Dr Sourish Banerjee (USQ)
PROJECT AIM:	This project aims to find the optimum topology of a large reinforced concrete box culvert (RCBC) under SM1600 loads using finite element analysis and Solid Isotropic Material with Penalization (SIMP) method. A feasibility analysis will then be conducted to ascertain how much can be saved using the optimum design and how practical it would be to design, manufacture and install it in practice.
PROGRAMME:	Issue F, 16/10/14

1. Read and analyse literature relating to optimisation of reinforced concrete products to produce a literature review.
2. Generate 3D finite-element models of standard reinforced concrete large box culvert (RCBC) sizes and design them as per AS1597.2-2013 under SM1600 loads.
3. Use the Solid Isotropic Material with Penalization (SIMP) method to optimise the RCBC topology
4. Perform a feasibility analysis of the optimum box culvert to ascertain the level of efficiency and cost savings in designing, manufacturing and installing the optimised structure.

As time permits:

5. Re-analyse culverts including shear reinforcement and draw conclusions regarding its influence in the optimum design

AGREED:

Student: Juliana Sweeney _________________________ Date: ___/___/2014

Supervisor: Dr. Sourish Banerjee __________________ Date: ___/___/2014

APPENDIX B – FLEXANALYSIS.M

```
%ANALYSIS AND DESIGN FOR FLEXURAL STRENGTH
function flexanalysis(D,ntension,astbar,ncomp,ascbar)
%INPUTS:
% D=crown thickness in mm
% ascbar= diameter of compression bars in mm
% astbar= diameter of tension bars in mm
% ncomp= quantity of compression bars
% ntension= quantity of tension bars

clc;
fc=50;        %MPa
b=2400;       %mm
cover=35;     %mm
Es=200E3;     %Mpa
bars=[10,80;12,110;16,200;20,310;24,450;28,620;32,800
;36,1020;40,1260];

%calcs
[a a1]=find(bars==ascbar);
Asc=ncomp*bars(a,2);          %area of steel in
compression
[a a1]=find(bars==astbar);
Ast=ntension*bars(a,2);       %area of steel in tension

T=500*Ast*1E-3;               %tension force
gamma=1.05-0.007*fc;          %gamma
if gamma>0.85
    gamma=0.85;
end
if gamma<0.67
    gamma=0.67;
end
alpha2=1-.003*fc;             %alpha2
if alpha2>0.85
    alpha2=0.85;
end
if alpha2<0.67
    alpha2=0.67;
end

rep=1;                        %rep=1 while dn has not
been found

while rep==1
    for dn=0.1:0.01:D
    Cc=gamma*dn*b*alpha2*fc*1E-3;
    esc=0.003*(dn-(cover+ascbar/2))/dn;
```

```
    Cs=Es*esc*Asc*1E-3;
    a=Cc+Cs;
        if (Cc+Cs>=0.999*T) && (Cc+Cs<=1.001*T)
            dnfinal=dn;
            rep=0;
        end
    end
end

dn=dnfinal                          %dn receives dnfinal value
ku=dn/(D-cover-astbar/2)
if ku>=0.36
    fprintf('Ku is outside range. Ku must be <
0.36');
end
esc=0.003*(dn-(cover+ascbar/2))/dn
if esc >=0.0025
    fprintf('esc is outside range. esc must be <
0.0025');
end
d=D-cover-astbar/2;
dc=0.5*gamma*dn;
dsc=cover+ascbar/2;
Cc=gamma*dn*b*alpha2*fc*1E-3
Cs=Es*esc*Asc*1E-3
Mu=(Cc*(d-dc)+Cs*(d-dsc))/1E3
phiMu=0.8*Mu
formatSpec = 'Number of N%2.0f bars required for
compression: %2.0f\nNumber of N%2.0f bars required
for tension: %2.0f\n';
fprintf(formatSpec,ascbar,ncomp,astbar,ntension);
end
```

APPENDIX C – CHECKSHEAR.M

```
function strresult =
checkshear(Vstar,astbar,ntension,D)
% Checks shear requirements according to AS3600
%inputs:     Vstar = V* in N
%            astbar = diameter of tension bars in mm
%            ntension = quantity of tension bars in mm
%            D = beam depth in mm
clc;
%Constants:
fc=50;       %MPa
bv=2400;     %mm
cover=35;    %mm
Es=200E3;    %Mpa
bars=[10,80;12,110;16,200;20,310;24,450;28,620;32,800
;36,1020;40,1260];
phi=0.7;
Vus=0;
fsyf=500;    %using N12 as ligatures

[a a1]=find(bars==astbar);
Ast=ntension*bars(a,2);

%calcs
d0=D-cover-astbar/2;    %the distance from the extreme
compressive fibre to the ...
                        %...centroid of the most
tensile reinforcement
Vumax=0.2*fc*bv*d0

fcv=fc^(1/3);
if fcv>4
    fcv=4;
end

b1=1.1*(1.6-d0/1000);
if b1<1.1
    b1=1.1;
end
b3=1;
b2=1;
beta=b1*b2*b3;
Vuc=beta*bv*d0*fcv*(Ast/(bv*d0))^(1/3)

half_phi_vuc=0.5*phi*Vuc;
if Vstar<=half_phi_vuc
    strresult='No shear reinforcement is required';
end
```

```
Vumin=Vuc+0.1*sqrt(fc)*bv*d0;
if Vumin<(Vuc+0.6*bv*d0)
    Vumin=(Vuc+0.6*bv*d0);
end
Vumin
if Vstar>half_phi_vuc
    if Vstar<=phi*Vumin
        if Vstar<=phi*Vuc
            strresult='Minimum shear reinforcement is
required but may be waived.';
        else
            strresult='Minimum shear reinforcement is
required.';
            Asvmin=0.06*sqrt(fc)*bv*(0.5*D)/fsyf
        end
    end
end

if Vstar>phi*Vumin
    nshear=5;       %Quantity of N12 bars for shear
reinforcement
    Asv=110*nshear; %N12 design area=110mm2

    % Minimum Vus = Vusmin
    Vusmin=(Vstar-phi*Vuc)/phi
    if Vuc+Vusmin>=(Vumax)
        strresult='Design fails in shear. Increase
thickness';
    end
    s_vusmin=Asv*fsyf*d0/(Vusmin*tand(45))
    s=D/(nshear-1)

    %Maximum Vus
    Vusmax=Vumax-Vuc
    s_vusmax=Asv*fsyf*d0/(Vusmax*tand(45))

    formatSpec = 'Shear reinforcement is required.
Provide %2.0f-N12 bars at %2.0f mm spacings\n';
    strresult=fprintf(formatSpec,nshear,s);
end
end
```

APPENDIX D – FINALSCRIPT.M

```
function [finalresults,finalresulta,nodes] =
finalscript(lowestfill,
highestfill,leg,crown,nomspan,nomheight)
%inputs: all inputs in metres
clc;
ht=lowestfill;
finalresults=[];
finalresulta=[];

while ht<=(highestfill+1E-10) %for
ht=lowestfill:0.1:highestfill
    [Wdc Wfv Wcv Wlv Wfh Wah Wch Wlh Bc oheight crown
leg] = RCBC(ht,leg,crown,nomspan,nomheight);
    load_comb;
    [a b]=size(scomb);

%generate strand7 inputs for node creation
nodes=[0,0;0,oheight-crown/2;Bc-leg,oheight-
crown/2;Bc-leg,0];

%FIND WORST SYMMETRIC LOAD CASE
    worstsymloadcomb=max(scomb,[],1);      %This
would be the worst load comb

    %confirm it exists:
    exists=0;
    for counter=1:a
        result=scomb(counter,:)-worstsymloadcomb;
        ind=find(result);
            if isempty(ind);
                exists=1;
                worst=counter;
            end
    end

finalresults=[finalresults;ht,worst,(scomb(worst,:))]
;
ht=ht+0.1;

end %end while

%%%%%%%PLOT SYMMETRIC LOAD COMBINATIONS
[a1 b1]=size(finalresults);
maxsym=0;
for counter7=1:a1

plot(finalresults(counter7,3:(int32(Bc*1000+3))));
```

```
    %str7=strcat(num2str((counter7-1)*0.1), ' m of
fill');
    %gtext(str7)
    if finalresults(counter7,3)>maxsym
        maxsym=finalresults(counter7,3);
    end
    hold on
end
title('Symmetric load combinations - top of culvert')
xlabel('Width (mm)')
ylabel('Load (kPa)')
figure

for counter7=1:a1
    plot(finalresults(counter7,int32(Bc*1000+4:b1)));
    hold on
end
title('Symmetric load combinations - sides of
culvert');
xlabel('Height from top of culvert (mm)');
ylabel('Load (kPa)');

max2=max(finalresults(:,int32(Bc*1000+4):b1));
end
```

APPENDIX E – DEVLENGTH.M

```
%%%calculates development length of bar in tension
according to AS3600
%%%clause 13.1.2 for 50MPa concrete, fsy=500MPa
function
Lsyt=devlength(barqty,db,cover,widthmember,k1)
%INPUTS:
%barqty - quantity of bars
%db - diameter of bar
%cover - cover to reinforcement in mm
%widthmember - width over which quantity of bars will
be srpread
%k1=1.3 for a horizontal bar with more than 300mm of
concrete cast below it
%or k1=1 otherwise
fsy=500;
fc=50;
a=(widthmember-2*cover)/(barqty-1);
cd=min(a/2,cover);
k3=1-0.15*(cd-db)/db;
k2=(132-db)/100;
Lsyt=max(0.5*k1*k3*fsy*db/(k2*sqrt(fc)),29*k1*db);
end
```

APPENDIX F – TOP_RCBC4.M

```
%%%% A 99 LINE TOPOLOGY OPTIMIZATION CODE BY OLE
SIGMUND, JANUARY 2000 %%%
%%%% CODE MODIFIED FOR INCREASED SPEED, September
2002, BY OLE SIGMUND %%%

%Adapted by Juliana Sweeney - October 2014

function
[x,U]=top_rcbc4(span,legheight,volfrac,penal,rmin)
% INITIALIZE variables
%calculate nelx, nely
scale=25;
legt=350;
crown=400;
nelx=ceil((span/2+legt)/scale)
nely=ceil((legheight+crown)/scale)
x(1:nely,1:nelx) = volfrac;

%creating box culvert hole
passive(crown/scale+1:nely,1:nelx-legt/scale)=1;
x(crown/scale+1:nely,1:nelx-legt/scale)=0.001;

loop = 0;
change = 1.;
% START ITERATION
while change > 0.01
  loop = loop + 1;
  xold = x;
% FE-ANALYSIS
  [U]=FE(nelx,nely,x,penal);
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
  [KE] = lk;
  c = 0.;
  for ely = 1:nely
    for elx = 1:nelx
      n1 = (nely+1)*(elx-1)+ely;
      n2 = (nely+1)* elx   +ely;
      Ue = U([2*n1-1;2*n1; 2*n2-1;2*n2;
2*n2+1;2*n2+2; 2*n1+1;2*n1+2],1);
      c = c + x(ely,elx)^penal*Ue'*KE*Ue;
      dc(ely,elx) = -penal*x(ely,elx)^(penal-
1)*Ue'*KE*Ue;
    end
  end
% FILTERING OF SENSITIVITIES
  [dc]   = check(nelx,nely,rmin,x,dc);
% DESIGN UPDATE BY THE OPTIMALITY CRITERIA METHOD
```

```
  [x]     = OC(nelx,nely,x,volfrac,dc,passive);
% PRINT RESULTS
  change = max(max(abs(x-xold)));
  aux1=clock;
  disp([' It.: ' sprintf('%4i',loop) ' Obj.: '
sprintf('%10.4f',c) ...
       ' Vol.: '
sprintf('%6.3f',sum(sum(x))/(nelx*nely)) ...
        ' ch.: ' sprintf('%6.3f',change ) ...
        ' time: ' sprintf('%4i',aux1(4)) ' '
sprintf('%4i',aux1(5)) ' ' sprintf('%3.0f',aux1(6))])
% PLOT DENSITIES
colormap(gray); imagesc(-x); axis equal; axis tight;
axis off;pause(1e-6);
end

%%%%%%%%%% OPTIMALITY CRITERIA UPDATE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [xnew]=OC(nelx,nely,x,volfrac,dc,passive)
l1 = 0; l2 = 100000; move = 0.2;
while (l2-l1 > 1e-4)
  lmid = 0.5*(l2+l1);
  xnew = max(0.001,max(x-
move,min(1.,min(x+move,x.*sqrt(-dc./lmid)))));
    %added line for passive
  xnew(find(passive))=0.001;

  if sum(sum(xnew)) - volfrac*nelx*nely > 0;
    l1 = lmid;
  else
    l2 = lmid;
  end
end

%%%%%%%%%% MESH-INDEPENDENCY FILTER
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [dcn]=check(nelx,nely,rmin,x,dc)
dcn=zeros(nely,nelx);
for i = 1:nelx
  for j = 1:nely
    sum=0.0;
    for k = max(i-
floor(rmin),1):min(i+floor(rmin),nelx)
      for l = max(j-
floor(rmin),1):min(j+floor(rmin),nely)
        fac = rmin-sqrt((i-k)^2+(j-l)^2);
        sum = sum+max(0,fac);
        dcn(j,i) = dcn(j,i) +
max(0,fac)*x(l,k)*dc(l,k);
      end
    end
```

```
    dcn(j,i) = dcn(j,i)/(x(j,i)*sum);
  end
end

%%%%%%%%%% FE-ANALYSIS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [U]=FE(nelx,nely,x,penal)
[KE] = lk;
K = sparse(2*(nelx+1)*(nely+1), 2*(nelx+1)*(nely+1));
F = sparse(2*(nely+1)*(nelx+1),1); U =
zeros(2*(nely+1)*(nelx+1),1);
for elx = 1:nelx
  for ely = 1:nely
    n1 = (nely+1)*(elx-1)+ely;
    n2 = (nely+1)* elx   +ely;
    edof = [2*n1-1; 2*n1; 2*n2-1; 2*n2; 2*n2+1;
2*n2+2; 2*n1+1; 2*n1+2];
    K(edof,edof) = K(edof,edof) +
x(ely,elx)^penal*KE;
  end
end

%%%%%%%% DEFINE LOADS AND SUPPORTS

%%TOP LINE LOAD: this will be manually changed with
each simulation
lineload(1)=1;
for aux=1:nelx
    lineload(aux+1)=lineload(aux)+nelx;
end
F(2*lineload,1) = -1575E3;    %load in N/m

%%SIDE LINE LOAD: this will be manually changed with
each simulation
lineload2(1)=2*(nely+1)*(nelx)+1;
for aux=1:nely+1
    lineload2(aux+1)=lineload2(aux)+2;
end
F(1*lineload2,1) = -833E3;     %load in N/m

%%%%%%%% END DEFINE LOADS AND SUPPORTS

fixeddofs   =
union([1:2:2*(nely+1)],[2*(nelx+1)*(nely+1)]);
alldofs     = [1:2*(nely+1)*(nelx+1)];
freedofs    = setdiff(alldofs,fixeddofs);
% SOLVING
U(freedofs,:) = K(freedofs,freedofs) \ F(freedofs,:);
U(fixeddofs,:)= 0;
```

```
%%%%%%%%%% ELEMENT STIFFNESS MATRIX
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [KE]=lk
E = 34800E6;  %Young's Modulus in Pa
nu = 0.2;
k=[ 1/2-nu/6   1/8+nu/8 -1/4-nu/12 -1/8+3*nu/8 ...
   -1/4+nu/12 -1/8-nu/8  nu/6       1/8-3*nu/8];
KE = E/(1-nu^2)*[ k(1) k(2) k(3) k(4) k(5) k(6) k(7)
k(8)
                  k(2) k(1) k(8) k(7) k(6) k(5) k(4)
k(3)
                  k(3) k(8) k(1) k(6) k(7) k(4) k(5)
k(2)
                  k(4) k(7) k(6) k(1) k(8) k(3) k(2)
k(5)
                  k(5) k(6) k(7) k(8) k(1) k(2) k(3)
k(4)
                  k(6) k(5) k(4) k(3) k(2) k(1) k(8)
k(7)
                  k(7) k(4) k(5) k(2) k(3) k(8) k(1)
k(6)
                  k(8) k(3) k(2) k(5) k(4) k(7) k(6)
k(1)];
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%
% This Matlab code was written by Ole Sigmund,
Department of Solid         %
% Mechanics, Technical University of Denmark, DK-2800
Lyngby, Denmark.      %
% Please sent your comments to the author:
sigmund@fam.dtu.dk             %
%
%
% The code is intended for educational purposes and
theoretical details     %
% are discussed in the paper
%
% "A 99 line topology optimization code written in
Matlab"                  %
% by Ole Sigmund (2001), Structural and
Multidisciplinary Optimization,     %
% Vol 21, pp. 120--127.
%
%
%
% The code as well as a postscript version of the
paper can be                %
% downloaded from the web-site:
http://www.topopt.dtu.dk                   %
```

```
%
%
% Disclaimer:
%
% The author reserves all rights but does not
guaranty that the code is    %
% free from errors. Furthermore, he shall not be
liable in any event       %
% caused by the use of the program.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%
```

APPENDIX G – LOAD_COMB.M

```
% GENERATING LOAD COMBINATIONS FOR RCBCs

%Bc= overall outside width of culvert in m - this
input variable is
%passed from main program

clc;
%Load factors:
vfactors=[  1    1.4 0    1.8
            1    1.4 1.5  0
            1    0.9 0    1.8
            1    0.9 1.5  0
            1    0.9 0    0
            1    1.4 0    0];

hfactors=[  0.7 0.7 0    1.8
            0.7 1.4 0    1.8
            1.4 0.7 0    1.8
            1.4 1.4 0    1.8
            0.7 0.7 1.5  0
            0.7 1.4 1.5  0
            1.4 0.7 1.5  0
            1.4 1.4 1.5  0
            0.7 0.7 0    0
            0.7 1.4 0    0
            1.4 0.7 0    0
            1.4 1.4 0    0];

%Horizontal Loads:
for counter5=1:length(hfactors)

h(counter5,:)=hfactors(counter5,1)*Wfh+hfactors(c
ounter5,2)*Wah+hfactors(counter5,3)*Wch+hfactors(
counter5,4)*Wlh;
end

%Vertical Loads:
for counter4=1:length(vfactors)

v(counter4,:)=vfactors(counter4,1)*Wdc+vfactors(c
ounter4,2)*Wfv+vfactors(counter4,3)*Wcv+vfactors(
counter4,4)*Wlv;
end
```

```
%FROM COL=1 TO COL=Bc*1000+1 IT'S VERTICAL, FROM
Bc*1000+2 TO length(scomb) IT'S HORIZONTAL
%Symmetric Combinations:

scomb=[ v(1,:),h(1,:); v(1,:),h(2,:);
v(1,:),h(3,:); v(1,:),h(4,:);
        v(2,:),h(5,:); v(2,:),h(6,:);
v(2,:),h(7,:); v(2,:),h(8,:);
        v(3,:),h(1,:); v(3,:),h(2,:);
v(3,:),h(3,:); v(3,:),h(4,:);
        v(4,:),h(5,:); v(4,:),h(6,:);
v(4,:),h(7,:); v(4,:),h(8,:);
        v(5,:),h(9,:); v(5,:),h(10,:);
v(5,:),h(11,:); v(5,:),h(12,:);
        v(6,:),h(9,:); v(6,:),h(10,:);
v(6,:),h(11,:); v(6,:),h(12,:)];

%Asymmetric Combinations:
%one side:
acomb1=[v(1,:),h(9,:); v(1,:),h(10,:);
v(1,:),h(9,:); v(1,:),h(10,:);
        v(2,:),h(9,:); v(2,:),h(10,:);
v(2,:),h(9,:); v(2,:),h(10,:);
        v(3,:),h(9,:); v(3,:),h(10,:);
v(3,:),h(9,:); v(3,:),h(10,:);
        v(4,:),h(9,:); v(4,:),h(10,:);
v(4,:),h(9,:); v(4,:),h(10,:);
        v(5,:),h(9,:); v(5,:),h(10,:);
v(5,:),h(9,:); v(5,:),h(10,:);
        v(6,:),h(9,:); v(6,:),h(10,:);
v(6,:),h(9,:); v(6,:),h(10,:);];
%opposite side:
acomb2=[v(1,:),h(11,:); v(1,:),h(11,:);
v(1,:),h(12,:); v(1,:),h(12,:)
        v(2,:),h(11,:); v(2,:),h(11,:);
v(2,:),h(12,:); v(2,:),h(12,:);
        v(3,:),h(11,:); v(3,:),h(11,:);
v(3,:),h(12,:); v(3,:),h(12,:);
        v(4,:),h(11,:); v(4,:),h(11,:);
v(4,:),h(12,:); v(4,:),h(12,:);
        v(5,:),h(11,:); v(5,:),h(11,:);
v(5,:),h(12,:); v(5,:),h(12,:);
        v(6,:),h(11,:); v(6,:),h(11,:);
v(6,:),h(12,:); v(6,:),h(12,
```

APPENDIX H – 1815 RCBC DESIGN

The design process of an RCBC is iterative. The first structure size trialled had a leg thickness of 200 mm and crown thickness of 250 mm. The results from the load combination Matlab script `finalscript.m` (see appendix D for code) were as shown in Figure 4-5 and Figure 4-10. The loads on the top of the culvert are uniformly distributed over the entire top of culvert, as expected since the truncated prism model (see Section 3.2 for details) distributes the vehicle and construction loads uniformly over the top of the culvert. The other loads that act on top of the culvert are also uniformly distributed, namely the fill and the self-weight. There are 20 straight lines in Figure 4-5, each for a different fill height. The top line is the load when there is only 0.1 m of fill over the culvert, giving a load of 646.9 kPa for all values of x (width of culvert). The bottom lines are very close together, which makes it difficult to distinguish one from the other. However, the last line represents the load for the case when there is 2.0 m of fill over the culvert, giving 107.3 kPa.

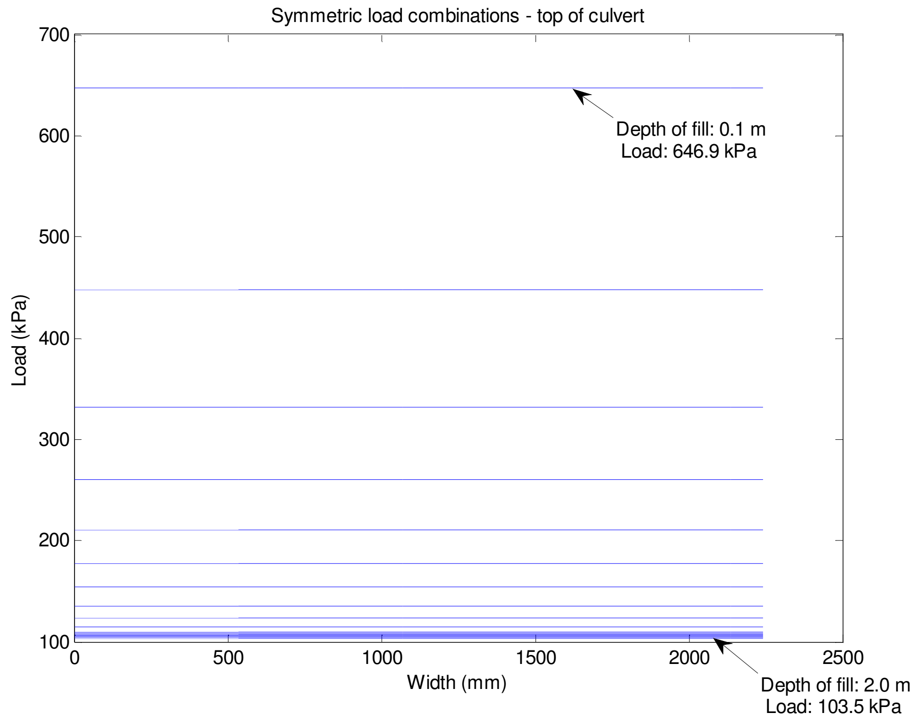

Figure H-1 – Loads on top of 1815 RCBC

The worst load case was that for the least amount of fill, conservatively chosen to be 0.1 m in this study (refer Section 3.2.3). The worst load combination was the vertical load number 1 (see Figure 4-1) and horizontal load number 10 (see Figure 4-2). This is expected since these combinations have the highest load factors.

The horizontal loads have a different form to the vertical loads. They increase from the top of culvert up to 0.5 m below the top of culvert, from which they continue uniformly. That is because the only horizontal load which is uniformly distributed is the horizontal live load W_{LH}. The compaction load W_{AH}, as dictated in AS1597.2-2013, increases up to 0.5 m below the top of the culvert, then remains constant up to 1.5 m below the top of the culvert, then decreases linearly up to 2.0 m below the top of the culvert (see Section 3.2.9 for details).

The horizontal fill load W_{FH} is also non-uniform since it varies with the depth below the culvert. Because of these non-uniformities, the shape of the horizontal load graphs differs from the vertical load graphs.

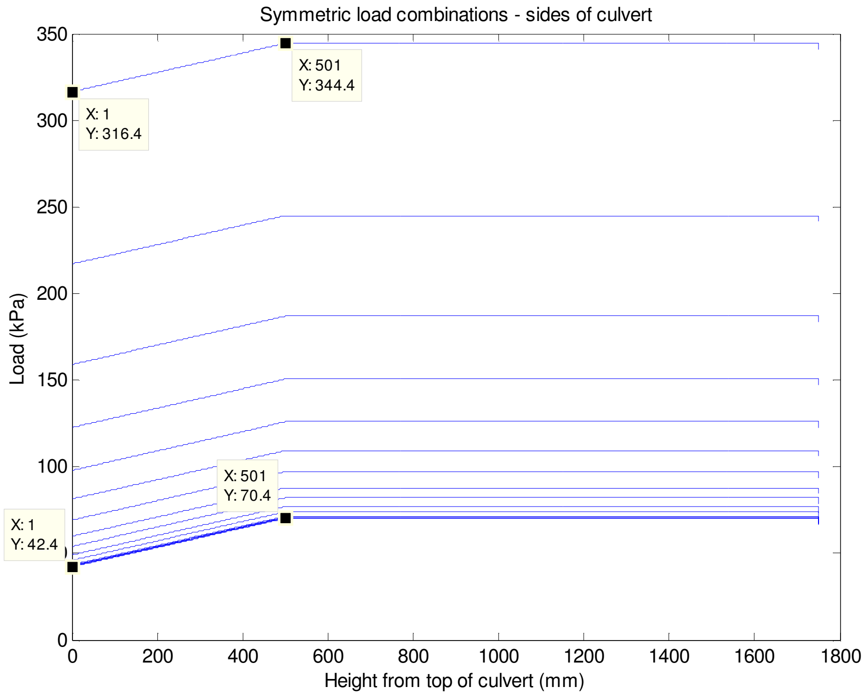

Figure H-2 – Loads on the side of 1815 RCBC

The 1815 RCBC is then modelled in Strand7 and the worst load case (for 0.1 m fill) is applied. The bending moment diagram and shear force diagram are then found as per Figure 4-11 and Figure 4-12.

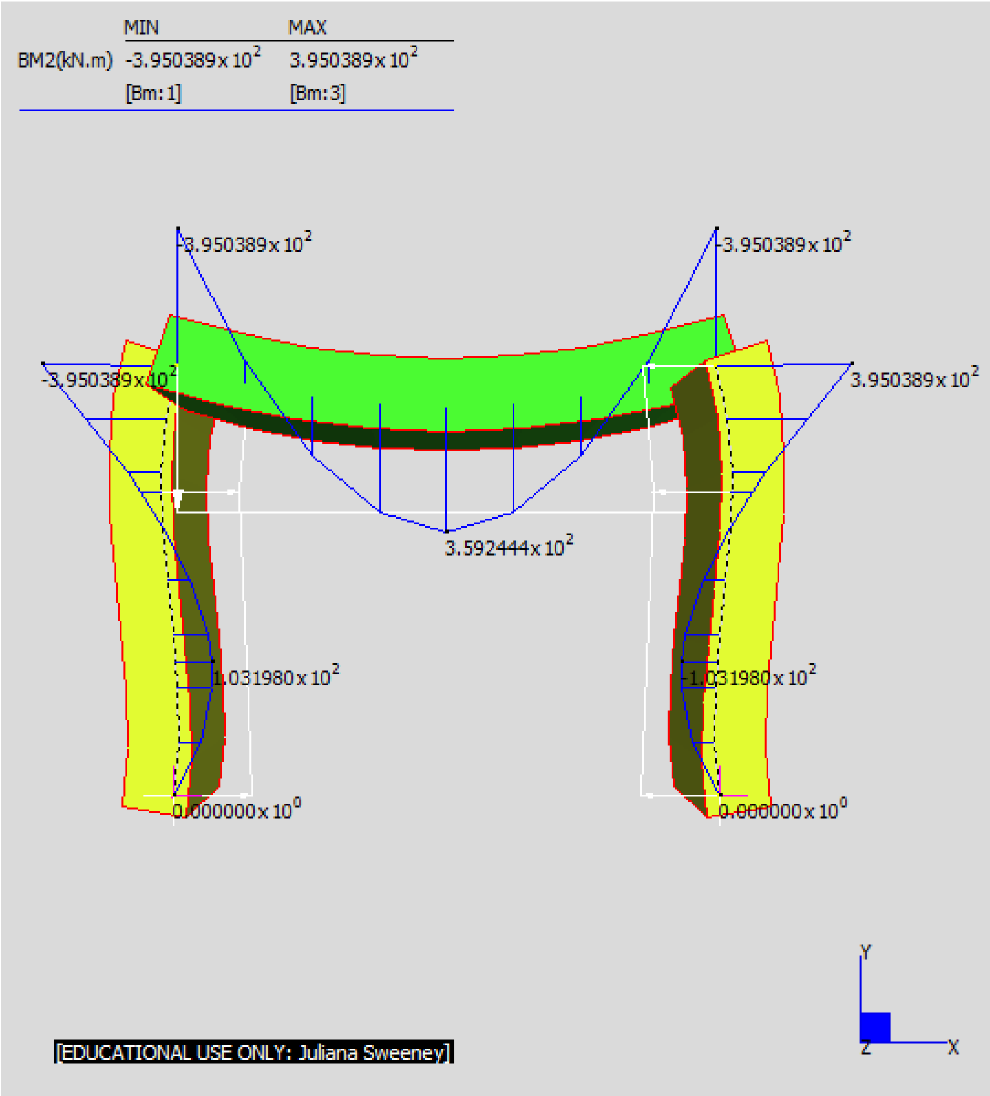

Figure H-3 – Bending moment diagram for 1815 RCBC

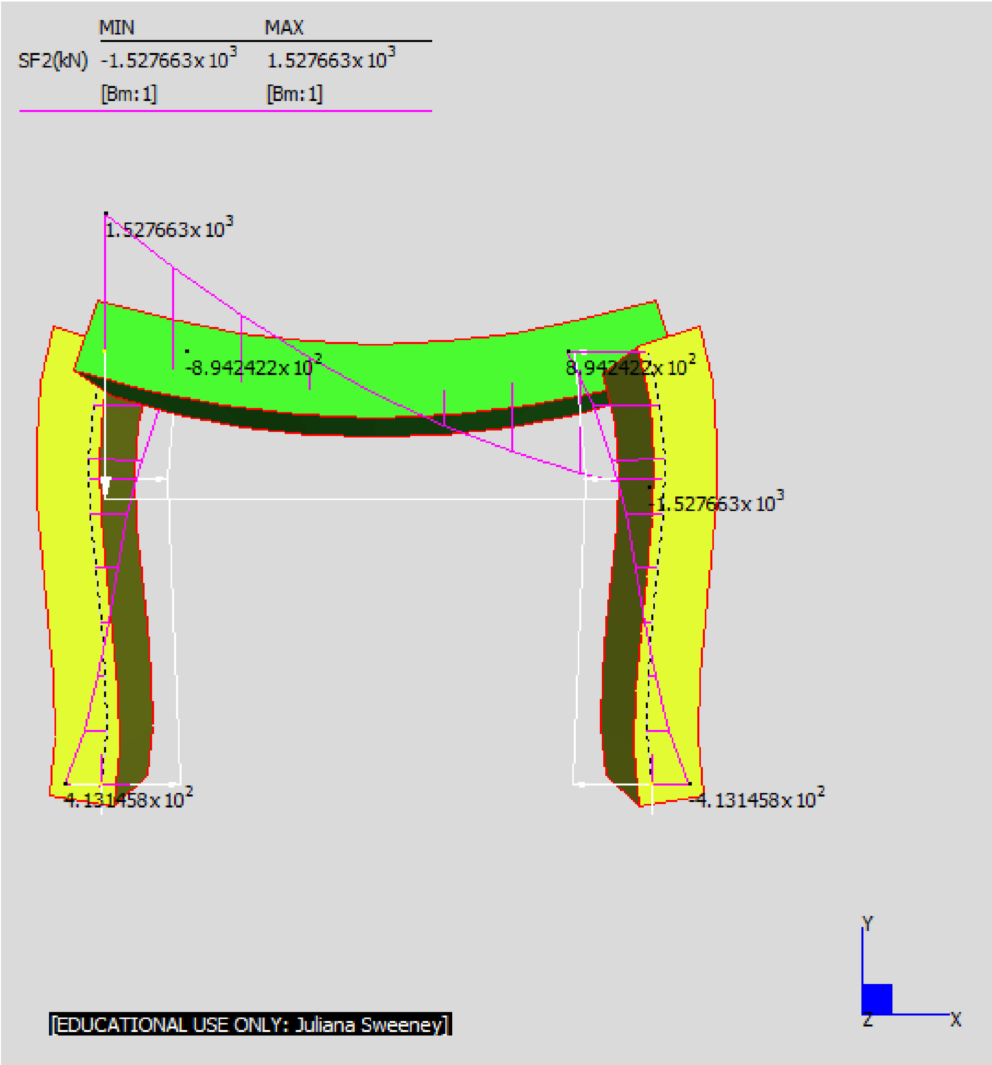

Figure H-4 – Shear force diagram for 1815 RCBC

The next step is to perform a flexibility and shear analysis to determine the suitable reinforcement for the unit, using the Matlab scripts `flexanalysis.m` and `checkshear.m`, as described in Section 3.3.5 - Non-optimised Culverts.

For the middle of the crown, M*=359.2 kNm (Figure 4-11) and by running `flexanalysis.m` with 16-N20 bars as tensile reinforcement and 5-N12 bars as compression reinforcement, the results are:

```
dn =   35.4800
ku =    0.1731
esc =  -4.6674e-04
Cc =   2.5333e+03
Cs =  -51.3416
Mu =  479.4426
phiMu =  383.5541
Number of N12 bars required for compression:  5
Number of N20 bars required for tension: 16
```

This means that:

- the section is under-reinforced since k_u<0.36;
- the compressive steel has not yielded since esc < 0.0025;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

The same procedure is utilised for the end of the crown, the top of the leg and the bottom of the leg. The shear is checked as described in Section 3.3.5 using the Matlab program `checkshear.m`. The development length of all bars is checked using the Matlab function `devlength.m` (see Matlab code in Appendix E) as per described in Section 3.3.6.

<u>Flexure and shear analysis for end of the crown</u>

The design moment at the end of the crown is M*=395.1 kNm (see Figure 4-11). By running `flexanalysis.m` for 24-N20 bars in tension and 11-N20 bars in compression, the results are:

```
dn =   49.5300
ku =    0.3195
esc =   2.7438e-04
Cc =   3.5364e+03
Cs =  187.1266
Mu =  507.4264
phiMu =  405.9412
```

```
Number of N20 bars required for compression: 11
Number of N20 bars required for tension: 24
```

This means that:

- the section is under-reinforced since k_u<0.36;
- the compressive steel has not yielded since esc < 0.0025;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

According to AS3600-2009 clause 8.2.5 (a), if $V^* \leq 0.5\phi V_{uc}$ no shear reinforcement is required except where the overall depth of the beam exceeds 750 mm, in which case minimum shear reinforcement shall be provided. Also according to the same standard, clause 8.2.5 (ii), if $V^* \leq \phi V_{uc}$ the minimum shear reinforcement requirements may be waived. Therefore, a shear check is required to check if shear reinforcement is necessary. The design shear force at the end of the crown is V*=1527.7 kN (see Figure H-4). By running `checkshear.m` for the 18-N20 bars in tension, the results are:

```
>> checkshear(1527.7E3,20,18,250);
Vumax =     4920000
Vuc =   6.2490e+05
Vumin =   9.7280e+05
Vusmin =   1.5575e+06
s_vusmin =   32.5757
s =   31.2500
Shear reinforcement is required. Provide 9-N12 bars at
31 mm spacings
```

This means that $V^* > \phi V_{u.\min}$ and shear reinforcement is to be provided as per AS3600-2009 clause 8.2.10. In this case, 9-N12 bars at 31 mm spacings would have to be provided for an extent of D=250 mm. However, this is not possible because the actual space between two N12 bars at 31 mm spacings is 19 mm measured from the outside of the bars. That is smaller than most maximum aggregate sizes in 50MPa concrete, which is 20 mm. By having bars too close together, it may impede the passage of the aggregate causing flow problems while casting. The minimum space between bars should not interfere with the casting procedure, and therefore this culvert would need to have its sections increased to better deal with the shear forces imposed on it.

<u>Flexure and shear analysis for top of the leg</u>

The design moment at the top of the leg is the same as the one at the end of the crown, as expected, and is M*=395.1 kNm (see Figure 4-11). However, this section is thinner since the leg is 220 mm and the crown is 250 mm. By running `flexanalysis.m` for 24-N20 bars in tension and 22-N20 bars in compression, the results are:

```
dn =   48.2700
ku =    0.3114
esc =   2.0323e-04
Cc =   3.4465e+03
Cs =  277.2082
Mu =  506.4705
phiMu =  405.1764
Number of N20 bars required for compression: 22
Number of N20 bars required for tension: 24
```

This means that:

- the section is under-reinforced since k_u<0.36;
- the compressive steel has not yielded since esc < 0.0025;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the top of the leg is V*=894.2 kN (see Figure H-4). By running `checkshear.m` for the 22-N20 bars in tension, the results are:

```
>> checkshear(894.2E3,20,22,220);
Vumax =     4200000
Vuc =   6.1417e+05
Vumin =   9.1116e+05
Vusmin =   6.6325e+05
s_vusmin =   43.5354
s =    44
Shear reinforcement is required. Provide 6-N12 bars at
44 mm spacings
```

This means that $V^* > \phi V_{u.\min}$ and shear reinforcement is to be provided as per AS3600-2009 clause 8.2.10. In this case, 6-N12 bars at 44 mm spacings would have to be provided for an extent of D=220 mm.

<u>Flexure and shear analysis for bottom of the leg</u>

The design moment at the bottom third of the leg is M*=103.2 kNm (see Figure 4-11). By running `flexanalysis.m` for 16-N12 bars in tension and 0 bars in compression, the results are:

```
dn =   12.3300
ku =    0.0775
esc =   -0.0079
Cc =  880.3620
Cs =     0
Mu =  136.1784
phiMu =  108.9427
Number of N20 bars required for compression:  0
Number of N12 bars required for tension: 16
```

This means that:

- the section is under-reinforced since k_u<0.36;
- $\phi M_u > M^*$ is suitable and so is the chosen reinforcement

A shear check is required to check if shear reinforcement is necessary. The design shear force at the bottom of the leg is V*=413.2 kN (see Figure H-4). By running `checkshear.m` for the 14-N20 bars in tension, the results are:

```
Vumax =      4200000
Vuc =    5.2828e+05
Vumin =    8.2526e+05
ans =Minimum shear reinforcement is required.
```

This means that $0.5\phi V_{uc} < V^{*} \leq \phi V_{u.\min}$ and minimum shear reinforcement $A_{sv.\min}$ is to be provided as per AS3600-2009 clause 8.2.8.

However, when the topology optimisation Matlab program is applied to the 1815 RCBC with the following input parameters:

span in mm = 1800
leg height in mm = 1500
volume constraint = 0.25
penalisation factor = 3
filter size divided by the element size (r_{min}) = 1.5

it yields the result shown in Figure 5-1. It can be seen that the only areas with white spaces (voids) are located in the bottom of the leg. That is the area where there is less bending moment in the RCBC and that also requires less reinforcement. The load on the crown is much larger than the load on the leg, and that means that the crown will not have voids, since the material in the crown is working hard to support the loads.

Figure H-5 – Topology optimisation result for of 1815 RCBC

This simulation took 99 seconds and required 135 iterations. The optimised culvert would look like shown in Figure H-6. It can be seen in detail 1 that there is no space to insert bars in the region of the voids, since the cover to reinforcement on either side of the bar needs to be 35 mm. Therefore, for topology optimisation to be performed on the 1815 RCBC, the design domain (crown thickness and leg thickness) will have to be increased to generate results that can be achieved in practice.

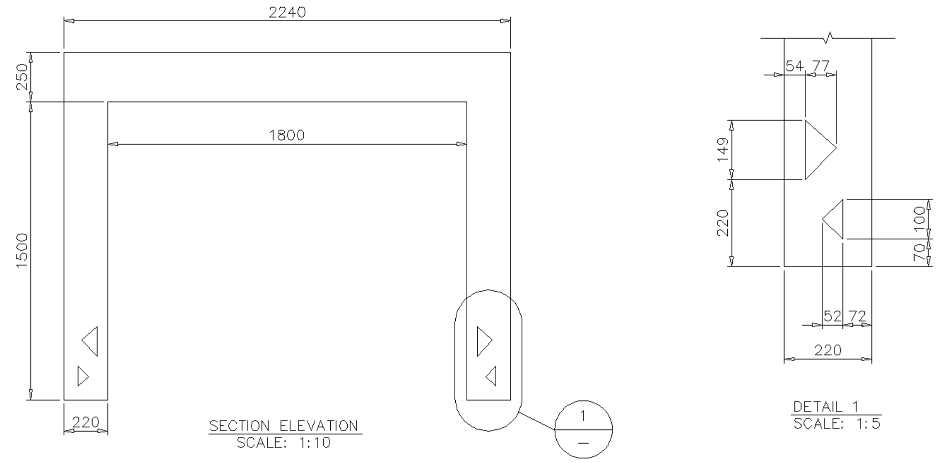

Figure H-6 – Optimised 1815 RCBC